Social welfare functions in optimizing climate-economy models:
A critical review

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Abstract

Social welfare functions (SWFs) are used in welfare-maximizing climate-economy models as objective functions to identify ‘optimal’ climate policies, to quantitatively compare alternative policies, and to characterize the costs of achieving alternative normative constraints. This paper reviews the main SWFs that have been applied in climate policy analyses according to the welfare maximization paradigm. We find that some of them are internally inconsistent (e.g., they lead to the summation of dimensionless and dimensional parameters) and/or produce counterintuitive rankings (e.g., they assign higher welfare to a low-growth than to a high-growth consumption path). We further find that determining the present value of future economic output or consumption paths is closely linked to the index number problem, which implies that no ‘correct’ calculation method exists. Different calculation methods applied in the past may lead to cost estimates of climate policy targets that differ by several orders of magnitude. Even the internally consistent SWFs are generally inconsistent with each other as they aggregate differently across time, population groups, and possible states of the world. We apply these theoretical findings to provide recommendations for the consistent use of SWFs in climate policy analyses according to the welfare maximization paradigm, and to review the application of SWFs in recent climate policy analyses. We find that some results of these analyses are artifacts caused by the inappropriate choice of SWFs. In summary, our findings indicate that the specification and combination of SWFs in climate policy analyses require considerably more caution than has been exercised in the past.

Keywords: climate change, integrated assessment, social welfare function, objective function, growth discounting, index number problem, DICE model, FUND model

Abbreviations: CRRA – constant relative risk aversion; DU – discounted utility; GHG – greenhouse gas; GMT – global mean temperature; GWP – gross world product; PDF – probability density function; PV – present value; PVC – present value of consumption; PVO – present value of output; SWF – social welfare function

JEL classification: C43, D60, D81, D90, O41, Q54
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1 Introduction

Simple climate-economy models based on a welfare maximization paradigm are still being used for a variety of climate policy analyses. Two prominent examples for this type of models are DICE (Nordhaus, 1994) and FUND (Tol, 1999). The DICE model, in particular, has frequently been adapted by researchers to investigate a wide variety of scientific and policy questions that are beyond the scope of the original model.

The ultimate goal of climate policy analysis according to the welfare maximization paradigm is to assess alternative policies according to a predefined social welfare function (SWF). A SWF is an algebraic formulation that assigns numerical social utility to each possible social state. In the context of this paper, we use the term ‘social welfare function’ to refer to any mathematical formulation that assigns a numerical value to a stream of economic output and/or consumption derived from a global climate-economy model. All models considered assume that individual utility is determined by a single economic good, and that all individuals can be characterized by the same utility function.

Ideally a SWF would be derived from the revealed preferences of the individuals concerned. However, Arrow’s Impossibility Theorem (Arrow, 1951) shows that there is no unique method for aggregating individual preferences into social preferences. Even if such an aggregation was theoretically possible, it would not be practical in the case of climate policy analysis, which will significantly affect future generations who cannot reveal their preferences today. For that reason, the SWFs applied in the climate change context are ‘synthetic’, constructed with the aim to reflect the implicit or explicit preference structure of current decision-makers.

A SWF is based on individual utilities, which can be regarded as either ordinal or cardinal (i.e., interval-scaled or rational-scaled). Ordinal utilities are sufficient for identifying the optimal policy alternative in a deterministic context involving a single individual (or several identical individuals). In contrast, the aggregation of utilities across different individuals and/or different possible states of the world (in a probabilistic analysis) requires the underlying utilities to be cardinal. The same is true for SWFs that are used to compare alternative policies quantitatively.

We note that some economists reject the concept of cardinal utilities because such utilities cannot be solely derived from observations of actual behaviour (see, e.g., Arrow, 1951). The main categories of SWFs that have been used by climate-economy models for assessing the costs and benefits associated with alternative climate policies are discounted utility of consumption (DU), present value of consumption (PVC), and present value of economic output (PVO), and the particular SWFs has been defined for different assumptions regarding time discounting.

This paper reviews the application of social welfare functions in welfare-optimizing climate-economy models. The maximization of intertemporal welfare has been criticized, among others, for its neglect of the allocation of rights, its implicit assumption that intergenerational compensation is actually feasible, the assumption of full substitutability between market commodities and environmental goods and services, its inability to account reliably for deep uncertainty or catastrophic outcomes, and the weak empirical basis of widespread practices such as assuming representative agents and decision-makers that maximize international or intertemporal welfare, applying logarithmic utility, and exponential time discounting (Lind et al., 1982; Taylor, 1982; Lind, 1995; Lind & Schuler, 1998; Spash, 2002; Azar & Lindgren, 2003; DeCanio, 2003; Yohe, 2003; Gowdy, 2005). The focus of this paper, in contrast, is on potential logical and methodological inconsistencies associated with the application of various SWFs, which has received little
attention so far. We emphasize that our particular focus does not intend to disregard the more fundamental criticism of the uncritical application of the welfare maximization paradigm to the climate change problem.

This paper is structured as follows. The first part is a theoretical analysis of several SWFs that have been used in applications of the DICE and FUND models, two of the most widely used optimizing climate-economy models. Sect. 2 presents the SWFs investigated in this paper; Sect. 3 discusses potential internal inconsistencies of individual SWFs; and Sect. 4 examines several inconsistencies between different internally consistent SWFs. Based on the results of these theoretical sections, Sect. 5 presents recommendations for the application of SWFs in climate policy analysis based on the welfare maximization paradigm, and reviews the application of SWFs in selected policy analyses with the DICE model. Sect. 6 concludes the paper.

2 Notation and definitions

The following notation is used throughout this paper:

- \( Y \geq 0 \) economic output
- \( C \geq 0 \) consumption
- \( I \geq 0 \) investment
- \( L > 0 \) population
- \( \theta \geq 0 \) intertemporal elasticity of substitution
- \( g \) actual growth rate of per capita consumption
- \( \dot{g} \) assumed growth rate of per capita consumption
- \( \rho \geq 0 \) pure rate of time preference (aka ‘utility discount rate’)
- \( r \) social discount rate
- \( s \geq 0 \) investment (or savings) rate

These variables may be supplemented with a time index \( t \) (e.g., \( L_t \) denotes population in year \( t \)), whereby \( t = 0 \) refers to the present year and \( t = T \) to the final year of a time series. If such an index is missing, the respective variable is assumed to be constant over time. \( X_{uv} \) notes the stream of variable \( X \) from time \( u \) to time \( v \) (assuming \( u \leq v \)). For notational convenience, we assume the value of the ‘empty product’ to be one, i.e., \( \prod_{t=1}^{0} = 1 \).

The objective function most commonly applied in optimizing climate-economy models is utility from consumption, assuming a constant relative rate of risk aversion (CRRA; see DeCanio, 2003, Table 2.4). CRRA utility functions are also known as CIES (constant intertemporal elasticity of substitution) utility functions. A CRRA utility function for a single agent and a single point in time is defined by the time-invariant ‘felicity function’

\[
U_{\theta}(C) = \begin{cases} 
\frac{C^{1-\theta}}{1-\theta}, & \text{if } \theta \neq 1 \\
\ln C, & \text{if } \theta = 1
\end{cases}
\]

(1)

\( \theta \) is a measure of the curvature of the utility function; it is variably denoted as intertemporal elasticity of substitution, rate of relative risk aversion, or absolute value of the elasticity of the
marginal utility of per capita consumption. The values determined by the felicity function do not correspond to any ‘real world’ units; their unit is sometimes denoted as ‘util(s)’.

When multiple agents are concerned, the utilitarian approach defines social welfare as the (weighted) sum of individual utilities. However, the lack of a lower bound in the logarithmic utility function (i.e., for \( \theta = 1 \)) can lead to counter-intuitive results when utility is aggregated over multiple agents. For instance, the complete loss of income of a single agent (associated with infinite negative utility) cannot be counterbalanced by any finite income increase of all other agents.

DICE and other globally aggregated models that consider only one ‘representative’ agent equate total utility with the utility of per capita consumption multiplied by the population size (Nordhaus & Boyer, 2000, App. E):

\[
U_{\theta}(C, L) = L \cdot U_{\theta}(C).
\]

However, any comparison of social welfare across different population scenarios involves essential value judgements. In addition, we show in Sect. 3.2 that Eq. 2 produces arbitrary results if it is applied to different population scenarios.

In Eq. 3 to Eq. 8, we define six SWFs that have been used for comparing alternative policy strategies in connection with the DICE model.\(^1\) These SWFs take a finite output or consumption stream (expressed in currency, such as dollars) as input and calculate a scalar welfare value (expressed either in currency or in arbitrary ‘utils’) as output. All SWFs considered in this paper are defined as the discounted intertemporal sum of the welfare in each time step, and they assume deterministic future discount rates.

The definition of the SWFs below includes additional parameters that reflect social value judgements about time preference (\(\rho\)) and the distribution of wealth within generations (\(\theta\)). There is a wide range of literature on the most appropriate values for these parameters in economic models of climate change and on the question of discounting in general (Lind et al., 1982; Arrow et al., 1996; Nordhaus, 1997; Heal, 1997; Portney & Weyant, 1999; Toth, 2000; Howarth, 2003; Newell & Pizer, 2004). The standard value for \(\theta\) in economic models of climate change is unity (Arrow et al., 1996; DeCanio, 2003). The corresponding logarithmic (or Bernoullian) utility function is also applied in the DICE models.

There is more disagreement on appropriate values for \(\rho\), and on the question whether this parameter should be constant over time. While declining rates of pure time preference cause time inconsistency, the consideration of uncertainties about future discount rates can lead to decreasing expected discount rates without causing time-inconsistent behaviour (Newell & Pizer, 2004). The default value in DICE-94 is \(\rho = 3\%/yr\) (Nordhaus, 1994, p. 104), the original DICE-99 model assumes that \(\rho\) declines over time, from \(\rho = 3\%/yr\) in 1995 to \(\rho = 1.25\%/yr\) in 2335 (Nordhaus & Boyer, 2000, pp. 15–16), and the adaptation of DICE-99 applied in Yohe et al. (2004) assumes \(\rho = 0\%/yr\). The findings in this paper are largely independent of the choice of a constant or declining pure rate of time preference.

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\(^1\)The DICE model is a Ramsey–Cass-Koopmans optimal-growth model of the world economy in which a central planner maximizes intertemporal welfare subject to certain constraints by determining the optimal division of economic output over time into consumption, investment, and emissions abatement.
DU \text{DICE}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}) = \sum_{t=0}^{T} \frac{L_t \cdot \ln(C_t/L_t)}{\prod_{t'=1}^{t} (1 + \rho_{t'})} \quad (3)

\text{PVC \text{DICE}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}) = \sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t} (1 + \rho_{t'}) \cdot \frac{C_{t'}}{L_{t'}}} = \frac{C_0}{L_0} \sum_{t=0}^{T} \frac{L_t}{\prod_{t'=1}^{t} (1 + \rho_{t'})}} \quad (4)

\text{PVC_{end}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}, \theta) = \sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t} (1 + \rho_{t'} + \theta \cdot \tilde{g})}} \quad (5)

\text{PVC_{ex}(C_{0\cdots T}; \rho_{1\cdots T}, \theta, \tilde{g}) = \sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t} (1 + \rho_{t'} + \theta \cdot \tilde{g})}} \quad (6)

\text{PVO_{Yohe}(Y_{0\cdots T}, L_{0\cdots T}) = \sum_{t=0}^{T} \frac{Y_t}{\prod_{t'=1}^{t} (1 + \ln \frac{C_{t'}}{L_{t'}} - 1)}} \quad (7)

\text{PVO_{ex}(Y_{0\cdots T}; \rho_{1\cdots T}, \theta, \tilde{g}) = \sum_{t=0}^{T} \frac{Y_t}{\prod_{t'=1}^{t} (1 + \rho_{t'} + \theta \cdot \tilde{g})}} \quad (8)

DU \text{DICE} describes the logarithmic utility of consumption based on ‘classic’ utility discounting at the rate of pure time preference. This utility function is used as objective function in the original DICE-99 model (Nordhaus & Boyer, 2000, p. 181).  

The other SWFs express welfare in monetary units (i.e., currency). They apply some variant of growth discounting, which focuses on the social marginal utility of consumption today compared with consumption in the future and represents the ‘classical’ approach to time discounting (Arrow et al., 1996; Nordhaus, 1997; Heal, 1997; Tol, 1999; Toth, 2000). Growth discounting is based on findings about optimal savings in an idealized economy by Ramsey (1928). The conventional formula for social time preference, also known as the ‘Ramsey growth discounting rule’, is \( r = \rho + \theta g \), whereby \( r \) is often referred to as the social rate of time preference (SRTP). However, this formula is only an approximate solution of the Ramsey model (see App. A).

PVC \text{DICE} describes the present value of consumption as calculated in the original DICE-99 model, which applies a variant of growth discounting.

PVC_{end} describes the present value of consumption according to the conventional formulation of the ‘Ramsey growth discounting rule’. In PVC_{end}, the discount rate is determined based on the actual growth rate of per capita consumption in each year. This SWF has been widely applied in global economic models of climate change (see, e.g., Tol, 1999).

PVC_{ex} also describes the present value of consumption according to the Ramsey growth discounting rule. In contrast to PVC_{end}, the discount rate is determined based on an \textit{exogenously}
specified assumed growth rate of per capita consumption. This welfare measure has been used to determine the total welfare effects of climate policies in DICE-99 (see Nordhaus & Boyer, 2000, p. 127, and the rows Output!18 and 21 in dice99.xls).

PVO_{Yohe} describes the present value of economic output applying yet another variant of growth discounting. This SWF has been applied in a modified version of the DICE model (Yohe et al., 2004, and Yohe, pers. comm.) Neither $\theta$ nor $\rho$ are contained in the specification of PVO_{Yohe} since Yohe et al. (2004) assumes $\theta = 1$ and $\rho = 0$. \(^3\)

PVO_{ex} describes the present value of economic output according to the Ramsey growth discounting rule, whereby the discount rate is determined based on an exogenously specified assumed growth rate. PVO_{ex} is identical to PVC_{ex}, except that economic output is substituted for consumption. A special case of this welfare function (assuming $\theta = \rho_t = 0$) is applied in Fankhauser & Tol (2005), which apparently applies undiscounted GDP calculated by different versions of DICE-94.

3 Inconsistencies of individual welfare metrics

In this section, we discuss potential inconsistencies of individual welfare metrics defined in Sect. 2. Sect. 3.1 thoroughly examines inconsistencies and counterintuitive results related to the application of growth discounting in monetary welfare metrics, and Sect. 3.2 briefly discusses inconsistencies caused by the application of non-monetary welfare metrics across different population scenarios.

3.1 Growth discounting and the index number problem

The level of the social discount rate is often critical in determining optimal or efficient policy alternatives, in particular for long-term problems such as climate change. Therefore, debates about discounting have always occupied an important place in environmental policy and welfare economics. In this section, we analyze how the discounting schemes applied in the monetary SWFs defined above rank alternative consumption paths, and how they value the magnitude of the welfare difference between alternative paths.

For the sake of simplicity, this analysis assumes consumption paths with a constant growth rate, which are characterized as

$$C(t; C_0, g) = C_0 \cdot (1 + g)^t,$$

whereby $C_0$ denotes initial consumption at $t = 0$ and $g$ the rate of per capita consumption growth. In addition, we assume the following parameters to be constant: population ($L_t \equiv 1$).

\(^3\)Yohe et al. (2004, SOM pp. 1–2) states that “In this Policy Forum, the pure rate of time preference is set equal to zero. With an elasticity of marginal utility equal to unity, the social discount rate is simply the endogenously determined rate of annual growth of per capita consumption.” This text suggests that monetary values were discounted using the discounting scheme from PVC_{DICE} or PVC_{end} (the two are identical for $\theta = 1$ and $\rho = 0$). If this had indeed been the case, PVC would be identical across all consumption scenarios (see Sect. 3.1). However, the model code that was kindly provided by G. Yohe revealed that PVO_{Yohe} was actually used in determining discounted GWP and selecting the costs of alternative policies.
investment rate \((C_t/Y_t \equiv \text{const.})\), pure rate of time preference \((\rho_t \equiv \rho \geq 0)\), and elasticity of the marginal utility of consumption \((\theta_t \equiv \theta > 0)\).

Assuming constant population allows us to equate the growth rates for total consumption and for per capita consumption. Assuming a constant investment rate allows to equate the growth rates for economic output and for consumption. Assuming constant time preference, per capita consumption growth, and elasticity of the marginal utility of consumption leads to a constant social discount rate in the discounting schemes discussed here, which are all based on growth discounting. Furthermore, these simplifying assumptions allow the present analysis to be restricted to a single future time step. Given the time-separability of the SWFs considered here, results for individual points in time can be easily generalized to the whole time series.

Applying the welfare functions defined in Sect. 2 to the constant-growth consumption stream \(C(t; C_0, g)\) defined in Eq. 9, and equating \(Y_t\) with \(C_t\), results in the following welfare measures for an individual time step \(t\):

\[
\begin{align*}
\text{DU}_{\text{DICE}}(C(t; C_0, g), t; \rho) & = \frac{\ln (C_0 \cdot (1 + g)^t)}{(1 + \rho)^t} \quad (10) \\
\text{PVC}_{\text{DICE}}(C(t; C_0, g), t; \rho) & = \frac{C_0 \cdot (1 + g)^t}{((1 + \rho) \cdot (1 + g))^t} \\
& = \frac{C_0}{(1 + \rho)^t} \quad (11) \\
\text{PVC}_{\text{end}}(C(t; C_0, g), t; \rho, \theta) & = \frac{C_0 \cdot (1 + g)^t}{(1 + \rho + \theta \cdot g)^t} \quad (12) \\
\text{PVC}_{\text{ex}}(C(t; C_0, g), t; \rho, \theta, \tilde{g}) & = \frac{C_0 \cdot (1 + g)^t}{(1 + \rho + \theta \cdot \tilde{g})^t} \quad (13) \\
\text{PVO}_{\text{Yohe}}(C(t; C_0, g), t) & = \frac{C_0 \cdot (1 + g)^t}{(1 + \ln(1 + g))^t} \quad (14)
\end{align*}
\]

The first part of the analysis focuses on the ranking of consumption paths. We employ a monotonicity criterion that requires a SWF to assign higher welfare to a constant-growth consumption scenario with a higher growth rate (and thus consistently higher consumption) than to one with a lower growth rate. Formally, a discounted welfare metric \(DW\) fulfills the monotonicity criterion if

\[
\text{DW}(C(t; C_0, g), t) < \text{DW}(C(t; C_0, g + \Delta g), t)
\]

for all positive \(C_0, g, \Delta g, \text{ and } t\).

The motivation for this criterion is our firm conviction that the vast majority of climate policymakers seeking advice from optimal growth models would clearly prefer a policy scenario with consistently higher consumption (growth) over one with lower consumption (growth), everything else being equal. This assumption is also made implicitly in most climate policy analyses with optimal growth models. Fankhauser & Tol (2005), for instance, apply DICE to compare indirect climate impacts under different assumptions. Their main criterion is future loss in undiscounted GDP. The conclusions of their analysis would be reversed if they applied a welfare metric that
assigns higher welfare to scenarios with lower growth rates of consumption (or GDP) per capita. Therefore, we consider SWFs that violate the monotonicity criterion to be inconsistent with the preference structure of the target users and thus unsuitable for comparing alternative climate policies.

We find the following behaviour for the SWFs defined above:

**DU**

**DICE**, **PVC**\_ex, **PVO**\_ex and **PVO**\_Yohe always prefer the high-growth scenario over the low-growth scenario. In the case of **PVC**\_ex and **PVO**\_ex, this is true independent of the choice of \( \bar{g} \). Hence, these SWFs do fulfill the monotonicity criterion.

**PVC**\_DICE is insensitive to the consumption levels after the initial period. Hence, they do not fulfill the monotonicity criterion. We note that the “stationarity axiom” proposed by Koopmans (1960) is violated as well.\(^4\)

**PVC**\_end requires the distinction of three cases (see the proof below):

- For \( \theta < 1 + \rho \), higher welfare is assigned to the high-growth scenario. Therefore, the monotonicity criterion is fulfilled.
- For \( \theta = 1 + \rho \), the discounted welfare is independent of the growth rate of consumption. Therefore, the monotonicity criterion is not fulfilled. (For \( \theta = 1 \) and \( \rho = 0 \), **PVC**\_end is identical to **PVC**\_DICE.)
- For \( \theta > 1 + \rho \), higher welfare is assigned to the low-growth scenario. Therefore, the monotonicity criterion is not fulfilled.

For the proof of these statements, we assume \( C_0, \Delta g, \) and \( t \) to be positive.

\[
\text{PVC}_{\text{end}}(C(t; C_0, g), t; \rho, \theta) < \text{PVC}_{\text{end}}(C(t; C_0, g + \Delta g), t; \rho, \theta) \iff \frac{C_0 \cdot (1 + g)^t}{(1 + \rho + \theta \cdot g)^t} < \frac{C_0 \cdot (1 + g + \Delta g)^t}{(1 + \rho + \theta \cdot (g + \Delta g))^t} \iff \frac{1 + \rho + \theta \cdot (g + \Delta g)^t}{(1 + \rho + \theta \cdot g)^t} < \frac{1 + g + \Delta g}{1 + g} \iff \frac{\theta \cdot \Delta g}{1 + \rho + \theta \cdot g} < \frac{\Delta g}{1 + g} \iff \theta \cdot (1 + g) < 1 + \rho + \theta \cdot g \iff \theta < 1 + \rho
\]

In summary, we observe that **PVC**\_end fulfills the monotonicity criterion for some combinations of \( \theta \) and \( \rho \) but not for others. We further observe that the ‘switch value’ for the dimensionless parameter \( \theta \) depends linearly on the rate parameter \( \rho \), which is not

---

\(^4\)The stationarity axiom demands that if two sequences have the same start, then eliminating that common start and bringing the rest forward does not change their ranking.
dimensionless. The numerical value of $\rho$, for instance, is more than times larger when it is expressed per decade rather then per year. As a result, PVC_{end} may or may not fulfill the monotonicity criterion depending on the (arbitrary) choice of time step for its specification. The reason for this inconsistency is that growth discounting in PVC_{end} is based on an approximate solution rather than the exact solution of the Ramsey model (see App. A).

In summary, PVC_{DICE} and PVC_{end} violate the monotonicity criterion for many plausible combinations of the normative parameters, including $\theta = 1$ and $\rho = 0$. PVC_{end} is associated with further inconsistencies due to its lack of a solid theoretical foundation. Consequently, we consider these two SWFs unsuitable for evaluating and comparing alternative climate policies.

The monotonicity criterion discussed above only considers the ranking of alternative policies, i.e., it regards the welfare metrics as ordinal. Ordinal welfare metrics are sufficient for identifying cost-optimal strategies in a deterministic analysis, e.g., as objective function in a standard DICE optimization. In the remainder of this section, we analyze how the discounting schemes applied in the various monetary SWFs value the difference in present value (PV) between alternative policies. Monetary welfare metrics are, by definition, cardinal (i.e., rational-scaled). The crucial question addressed in this context is whether the use of different discount factors in the PV calculations for alternative policies is consistent, or not. This question reflects a yet unnoticed controversy in the integrated assessment modelling community, which is related to the well-known index number problem of economic statistics.

One opinion is expressed by the main developer of the DICE models: “The present values are computed using the base case discount factors.” (Nordhaus & Boyer, 2000, p. 127) and “In making welfare comparisons between two different policies, the same relative prices should be used to discount the future consumption streams that result from both policies. Thus, in constructing the comparison measures Total abatement cost of policy [...], we use the base case relative prices to discount both base case consumption and consumption under current policy.” (Nordhaus, 2001, p. 19). Consequently, the original DICE-99 model determines the monetary welfare associated with different policy alternatives by calculating PVC with the same discount factors (as in PVC_{ex}).

We are not aware of any explicit argument in favour of the opposite opinion (i.e., to use different discount factors for determining the PVs of alternative policies). However, several policy analyses with optimizing climate-economy models have applied SWFs that determine the discount factors for each policy option endogenously: FUND apparently applies PVC_{end} (Tol, 1999), and PVO_{Yohe} is applied in a variant of DICE-99 (Yohe et al., 2004, and Yohe, pers. comm.). Interestingly, the first authors of these two studies are fully aware of the potential problems associated with growth discounting when the discount rates are determined endogenously. They find that the discounting approach applied in PVC_{end} may lead to infinite expected damages from climate change (and thus infinite expected marginal benefits of mitigation) if there is the possibility for a catastrophic outcome (Tol, 2003), or if the analyst assumes an infinite time horizon (Yohe, 2003). As a consequence, Yohe (2003, p. 243) concludes that “we have added one more element to our list of reasons why it is inappropriate to use the expected value of discounted net benefits to judge mitigation policy”.

Let us briefly analyze the practical consequences of applying one or the other discounting approach before we consider the theoretical arguments. Table 1 compares the relative difference
Table 1: Difference in present value for various discounting schemes between two consumption streams growing at different rates.

<table>
<thead>
<tr>
<th></th>
<th>PVC&lt;sub&gt;DICE&lt;/sub&gt;</th>
<th>PVO&lt;sub&gt;Yohe&lt;/sub&gt;</th>
<th>PVC&lt;sub&gt;ex&lt;/sub&gt;&lt;br&gt;[&lt;span&gt;&lt;i&gt;g&lt;/i&gt; = 3%&lt;/span&gt;]</th>
<th>PVC&lt;sub&gt;ex&lt;/sub&gt;&lt;br&gt;[&lt;span&gt;&lt;i&gt;g&lt;/i&gt; = 0%&lt;/span&gt;]</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;span&gt;&lt;i&gt;g&lt;/i&gt; = 3%/yr&lt;/span&gt;</td>
<td>1000.0</td>
<td>1001.9</td>
<td>1000.0</td>
<td>1146.4</td>
</tr>
<tr>
<td>&lt;span&gt;&lt;i&gt;g&lt;/i&gt; = 0%/yr&lt;/span&gt;</td>
<td>1000.0</td>
<td>1000.0</td>
<td>878.6</td>
<td>1000.0</td>
</tr>
<tr>
<td>Difference</td>
<td>0%</td>
<td>-0.19%</td>
<td>-12.1%</td>
<td>-12.8%</td>
</tr>
</tbody>
</table>

In present value between two finite consumption streams starting at <span><i>C</i><sub>0</sub> = 100</span> and growing at either <span><i>g</i> = 3%</span> or <span><i>g</i> = 0%</span> over ten years for various discounting schemes. The <i>undiscounted</i> value of consumption for these two scenarios is 1146.4 and 1000.0, respectively (as in PVC<sub>ex</sub> for <span><i>g</i> = 0%/yr</span>). We assume <span><i>θ</i> = 1</span> and <span><i>ρ</i> = 0%/yr</span> since PVO<sub>Yohe</sub> is only defined for these parameter choices. As noted above, PVC<sub>DICE</sub> and PVC<sub>end</sub> determine the same PV for both consumption streams. PVO<sub>Yohe</sub> shows a small difference in PV between the two consumption streams but this difference is about 70 times smaller than the difference in undiscounted consumption. Hence, PVC<sub>ex</sub> is the only SWF considered here that adequately reflects the difference in (undiscounted) consumption between the two scenarios, and it does that largely independent of the exogenous choice of the growth rate, <span><i>g</i>.</span>

We will now discuss the discounting question more formally by analyzing the relationship between DU and PVC. Present value calculations at the micro-level (<i>e.g.</i>, for individual projects) determine how much money has to be put aside today to replicate a given stream of future returns under certain assumptions about future economic development. Put another way, PVC states the maximum amount that a rational person would be willing to pay for a contract that guaranteed to deliver a given utility stream from material consumption in the future. (Obviously, it is much more difficult to intuitively interpret PVC or PVO at the global level since putting aside the money equivalent to the future stream of gross world product is not possible.)

Let us consider a simple thought experiment. Imagine two rational individuals (A and B), each of whom has just agreed to a contract that will deliver a certain utility stream from future consumption (<span><i>U</i><sub>A</sub></span> and <span><i>U</i><sub>B</sub></span>) in return for a given payment now. Let us assume that A paid less for his contract than B (<i>i.e.</i>, <span>PVC<sub>A</sub><br>(<span>·</span>) < PVC<sub>B</sub><br>(<span>·</span>))). If you were offered the choice between the benefits from either of these contracts (without having to pay for the costs), which utility stream would you choose? If you knew that A and B had negotiated their contracts based on the same expectations about future economic growth rates (<i>i.e.</i>, <span>PVC<sub>A</sub><br>(<span>·</span>) = PVC<sub>B</sub><br>(<span>·</span>)), it would be safe to choose the higher valued contract (in this case: B’s contract), as this contract would also have higher (discounted) utility. However, if A and B had negotiated their contracts independently from each other, and B was assuming lower growth (and thus interest and discount) rates than A, B might have agreed to pay a higher price than A even if A’s contract delivers a higher utility (<i>i.e.</i>, <span><i>U</i><sub>A</sub> > <i>U</i><sub>B</sub>). In that case, any rational individual (including A and B) would assign a higher present value to the benefits of A’s contract than to those of B’s contract even though A paid less than B (<i>i.e.</i>, <span>PVC<sub>B</sub><br>(<span>U</span><sub>A</sub>) > PVC<sub>B</sub><br>(<span>U</span><sub>B</sub>) > PVC<sub>A</sub><br>(<span>U</span><sub>A</sub>) > PVC<sub>A</sub><br>(<span>U</span><sub>B</sub>)).

What are the implications of this thought experiment for the question debated here? Most importantly, PVC is a valid proxy for (discounted) utility only when the same discount factors are used in the PV calculations of all policy options. Otherwise, the ranking of policy options (<i>i.e.</i>, consumption streams) according to PVC may be inconsistent with the ranking according to DU.
The next question is, how much more worth are the benefits of A’s contract compared to those of B’s contract? Since A and B discount the difference in future benefits differently, they are likely to assign different present values to that difference (i.e., \( \text{PVC}_A(U_A) - \text{PVC}_B(U_B) \neq \text{PVC}_B(U_A) - \text{PVC}_B(U_B) \)). Hence, even in this simple example there is no unique metric for expressing the difference between the two utility streams in monetary terms.

The question of the ‘correct’ metric for expressing welfare differences between policies in monetary terms turns out to be closely related to the index number problem, which “can arise when an attempt is made to compare two [or more] sets of variables at two [or more] points in time using a single number since there are many different ways of aggregating variables into a single measure” (Pearce, 1986). The traditional index number problem is concerned with a situation in which there are \( N \geq 2 \) goods, \( I \geq 2 \) time periods, and 1 scenario. The relevant question is how to determine a single measure of the change of the overall quantity level between different periods. The problem discussed here, in contrast, is characterized by \( N \geq 2 \) time periods, 1 \( I \geq 2 \) scenarios, and 1 good. The relevant question then is how to determine a single measure of the difference of the overall quantity level (e.g., in present dollars) between different scenarios. In this context, the discount factors applied to future welfare losses correspond to the relative prices of different goods in the index number problem. The index number problem has concerned economists and statisticians since the 19\(^{th} \) century at least (Jevons, 1865), and it has long been known that no unique solution exists (see, e.g., Edgeworth, 1888, p. 347). Nevertheless, this problem is still inspiring research today (see, e.g., Diewert & Nakamura, 1993). While the index number problem does not have a unique solution, there is unanimous agreement that a single set of prices has to be used for a meaningful comparison of quantities between different periods. Applied to the question of interest here, this means that the same discount factors have to be used in PV calculations for all policy options considered.

The final question then is how to determine reasonable discount factors in the absence of a unique ‘correct’ method for choosing them? In our view, the most reasonable approach is to pick a ‘baseline’ policy scenario, and to calculate the discount factors based on the growth rates of this baseline scenario using PVC\(_{\text{ex}}\), as in the original DICE-99 model (Nordhaus, 2001, p. 19).\(^5\) Since the actual choice of the discount factors is relatively unimportant, we neglect in the present discussion that PVC\(_{\text{ex}}\) applies the discount factors based on the approximate solution of the Ramsey model rather than the exact solution (see App. A).

Summarizing the discussion above, all available evidence indicates that PV calculations should use the same discount factors for all policy options under consideration. Consequently, we consider PVC\(_{\text{DICE}}\), PVC\(_{\text{end}}\), and PVO\(_{\text{Yohe}}\) as unsuitable for comparing the welfare implications of alternative climate policies. PVO\(_{\text{Yohe}}\) is subject to two other problems as well. First, it is only defined for \( \rho = 0 \), which is widely considered unrealistically low (Arrow et al., 1996). Second, there is no theoretical basis for the logarithmic relationship between consumption growth rate and discount rate assumed in Eq. 14. Consequently, the only SWFs considered here that are consistent with the monotonicity criterion and with the other fundamental requirements for SWFs are DU\(_{\text{DICE}}\), PVC\(_{\text{ex}}\), and PVO\(_{\text{ex}}\). In the next two sections, we investigate whether these

\(^5\)We note that DICE-99 also computes PVC\(_{\text{DICE}}\), which is insensitive to the consumption levels after the initial period. However, the intention for computing PVC\(_{\text{DICE}}\) in DICE is not to distinguish between different policy alternatives but only to provide a monetary equivalent to which discounted utility in the no-policy case can be calibrated. (The actual objective function in DICE is DU\(_{\text{DICE-cal}}(\cdot) = \frac{\text{DU}_{\text{DICE}}(\cdot)}{\text{coefopt}_A} + \text{coefopt}_B, \text{ whereby coefopt}_A \) and coefopt\(_B\) are chosen so that DU\(_{\text{DICE-cal}}(\cdot) = \text{PVC}_{\text{DICE}}(\cdot) \) for an optimal strategy in the absence of climate damages.) The original DICE model does not report PVC\(_{\text{DICE}}\) for scenarios other than the baseline scenario.
internally consistent SWFs are also interchangeable with each other, i.e., whether they always produce the same ranking of alternative policies.

3.2 Utility calculation for different population scenarios

We stated in Sect. 2 that utility cannot be objectively compared across different population scenarios since such a comparison involves essential value judgements that are beyond the scope of economics. In addition to these philosophical arguments, we show here that the general practice of defining the total utility of several identical agents as the product of population size and per-capita utility provides arbitrary results if it is applied across different population scenarios.

Imagine that a given amount of total consumption, $C \cdot L$, is consumed by either $L$ or $k \cdot L$ identical individuals (assuming $k > 1$). The relationship for the total logarithmic utility of these two alternative scenarios is as follows:

$$DU_{\text{DICE}}(C, L) < DU_{\text{DICE}}(\frac{C}{k}, k \cdot L)$$

$$\iff L \cdot \ln C < k \cdot L \cdot \ln \frac{C}{k}$$

$$\iff k \cdot \ln k < (k - 1) \cdot \ln C$$

$$\iff k^{\frac{k}{k-1}} < C$$

We find that distributing the same total consumption $C \cdot L$ on a $k$ times larger population ($k > 1$) increases total utility if and only if

$$k^{\frac{k}{k-1}} < C$$

(15)

For instance, a doubling of the population ($k = 2$) increases total utility if $C > 4$. While this utility comparison requires that per capita consumption can be expressed as a pure number, the numerical value of $C$ depends on the units in which consumption is expressed.

We are not aware of any welfare-optimizing climate policy analysis that actually determines population growth rates endogenously. However, this approach was suggested by Fankhauser & Tol (2005), who motivated their analysis by listing three types of indirect climate impacts on human welfare, including “health and mortality impacts associated with more widespread diseases [that would] affect population growth” (p. 4). In contrast to this suggestion, their actual analysis with a modified version of DICE-94 apparently keeps population growth exogenous and implements the health effects of climate change by changing the accumulation of human capital instead (p. 15).

4 Inconsistencies between different welfare metrics

In this section, we investigate several potential inconsistencies between the internally consistent welfare metrics $DU_{\text{DICE}}$, $P_{\text{Vc}},$ and $P_{\text{Vo}}$. Sect. 4.1 investigates inconsistencies between
consumption-based and output-based welfare metrics; Sect. 4.2 investigates differences between monetary and non-monetary welfare metrics related to their aggregation of welfare across time; and Sect. 4.3 investigates differences between monetary and non-monetary welfare metrics related to their aggregation of welfare across possible states of the world.

4.1 Consumption-based vs. output-based welfare metrics

The optimal growth models considered here divide net economic output into consumption and investment in productive capital: \( Y = C + I \). Gross output also includes the costs of emissions abatement and climate change damages (i.e., net output is defined as gross output minus emissions abatement costs and climate change damages). The fraction of net output devoted to investment is denoted as the investment rate: \( s = \frac{I}{Y} = 1 - \frac{C}{Y} \). General equilibrium models such as DICE determine the investment rate endogenously. In the present discussion, we neglect the problem of converting investment into consumption equivalents (see, e.g., Lind & Schuler, 1998).

Our analysis comprises two parts, depending on whether an output-based or a consumption-based SWF is used as the objective function to be maximized. When \( \text{PVO}_{\text{ex}} \) instead of \( \text{DU}_{\text{DICE}} \) is used as objective function in DICE-99, the optimal (i.e., PVO-maximizing) policy is characterized by an —obviously unrealistic— investment rate of \( s = 100\% \) over the full time horizon. Since all economic output is used for investment \( (I = Y) \), none remains for consumption \( (C = 0) \), and utility becomes minus infinity \( (\text{DU}_{\text{DICE}} = -\infty) \). Thus the very policy that maximizes \( \text{PVO}_{\text{ex}} \) minimizes \( \text{PVC}_{\text{ex}} \) and \( \text{DU}_{\text{DICE}} \).

This simple example shows that consumption-based and output-based welfare metrics may rank alternative policies completely differently. From a practical point of view, the more relevant question is whether inconsistent rankings also occur for climate policies determined by DICE-99 in its ‘normal’ utility-maximizing mode. Fig. 1 shows (presumably) utility-maximizing decision strategies determined with a probabilistic version of DICE-99 (implemented in Analytica) for two different probabilistic climate constraints. The more and less stringent constraint limit the probability that global mean temperature (GMT) exceeds 2.5°C and 3.0°C above preindustrial levels, respectively, to 1% for a given probability density function of climate sensitivity. The top panel shows that the strategy for the less stringent 3.0°C constraint has higher consumption levels in the first 120 years but lower output levels in all but one period than the strategy for the 2.5°C constraint. Hence, the 3.0°C strategy has a higher \( \text{DU}_{\text{DICE}} \) and \( \text{PVC}_{\text{ex}} \) but a lower \( \text{PVO}_{\text{ex}} \) than the 2.5°C strategy. The explanation for the inconsistent ranking of the two policy strategies is shown in the bottom panel. The 3.0°C strategy is characterized by lower abatement rates (as expected) but also by significantly lower investment rates than the 2.5°C strategy.

A more detailed analysis reveals that the 3.0°C strategy shown in Fig. 1 does not actually maximize \( \text{DU}_{\text{DICE}} \) for that constraint. Even though the initial values of the decision variables were identical for both constraints, the gradient-based Analytica solver found the global optimum for the 2.5°C constraint but only a local utility optimum for the 3.0°C constraint. After changing the initial values, was able to find a decision strategy that satisfies the 3.0°C constraint with higher utility from consumption and higher output than the 2.5°C strategy. This finding highlights the importance of analyzing the ‘optimal’ results identified by model solvers carefully in order to avoid that local optima are misinterpreted as global optima. However, it does not challenge the main lesson to be learnt from this example: that two policy strategies determined
Figure 1: Decision strategies that maximize $\text{DU}_{\text{DICE}}$ for two probabilistic climate constraints. 
*Top:* Ratios of economic output and consumption.  
*Bottom:* Time trajectories of the two decision variables: investment (or savings) rate and abatement rate.
by DICE-99 in utility-maximizing mode may be ranked inconsistently by \(\text{DU}_{\text{DICE}}\) and \(\text{PVC}_{\text{ex}}\) compared to \(\text{PVO}_{\text{ex}}\). (This inconsistency between \(\text{PVC}_{\text{ex}}\) and \(\text{DU}_{\text{DICE}}\) occurs when comparing the optimal 2.5\(^\circ\)C strategy with the suboptimal 3.0\(^\circ\)C strategy but not with the optimal 3.0\(^\circ\)C strategy.)

### 4.2 Discounting dollars vs. discounting utils

The difference between discounting monetary values (e.g., consumption expressed in dollars or other currency) and discounting utility expressed in ‘utils’ is often blurred, as in the following quote from DeCanio (2003, p. 168): “According to the discounted utility formulation, an individual’s subjective time rate of discount \(\delta\) is determined by the relationship \(x = (1 + \delta)^t\), where the individual would be indifferent between $1 in consumption today and $x in consumption at time \(t\) in the future.” The wording of this sentence refers to discounted utility whereas the mathematical definition refers to discounting consumption.

We first analyze the relationship between the two discount rates for an agent that lives for two periods only. Since logarithmic utility becomes minus infinity if consumption in any time period is zero, we actually need to compare consuming additional $1 now with consuming additional \(\delta x = 1 + \delta\) in the next period. More generally, the logarithmic utility of consuming \(C_0\) at \(t = 0\) and \(C_1\) at \(t = 1\), discounted at rate \(r\), is

\[
\text{DU}(C_0, C_1; r) = \ln C_0 + \frac{\ln C_1}{1 + r}.
\]

Assuming a baseline consumption of \(C\) and denoting additional consumption as \(aC\) (e.g., \(a = \frac{\$1}{C}\) for an additional consumption of $1 at \(t = 0\)), we find the following relationships between the equivalent discount rates for utility, \(r\), and for consumption, \(\delta\):

\[
\text{DU}(C + aC, C; r) = \text{DU}(C, C + aC \cdot (1 + \delta); r) \quad (17)
\]

\[
\ln(C + aC) + \frac{\ln C}{1 + r} \iff \ln C + \frac{\ln (C + aC \cdot (1 + \delta))}{1 + r} \quad (18)
\]

\[
(1 + r) \cdot \ln (1 + a) \iff \ln (1 + a \cdot (1 + \delta)) \quad (19)
\]

\[
r = \frac{\ln (1 + a \cdot (1 + \delta))}{\ln (1 + a)} - 1 \quad (20)
\]

\[
\delta = \frac{(1 + a)^{1+r} - 1}{a} - 1 \quad (21)
\]

Fig. 2 shows that the two discount rates are very similar for marginal changes in baseline consumption (i.e., \(a \ll 1\)). For non-marginal consumption differences, however, discounting utility is inconsistent with the use of a single discount rate for consumption (and vice versa).

We show now that the difference between discounting consumption and utility may lead to inconsistent rankings of two consumption paths by \(\text{PVC}_{\text{ex}}\) and \(\text{DU}_{\text{DICE}}\). In this example, we are considering consumption paths that involve a one-time reduction in consumption relative
to a reference path with a constant growth rate. The relevant family of consumption paths is defined as follows:

$$C(t; g, a, t_a) = \begin{cases} (1 + g)^t, & \text{if } t \neq t_a \\ (1 + g)^t \cdot (1 - a), & \text{if } t = t_a \end{cases}$$

(22)

The two paths of specific interest here are identical except for $t = 0$ and $t = T$. $C(\cdot; g, a_0, 0)$ is characterized by a relative reduction in consumption of $a_0$ at $t = 0$ whereas $C(\cdot; g, a_T, T)$ involves a reduction of $a_T$ at $t = T$. We assume an elasticity of the marginal utility of per capita consumption of unity ($\theta = 1$) as well as constant population ($L_t \equiv 1$) and time preference ($\rho_t \equiv \rho$). These assumptions allow a simplified description of the two SWFs considered here:

$$\text{DU}_{\text{DICE}}(C_{0\ldots T}; \rho) = T \sum_{t=0}^{T} \frac{\ln C_t}{(1 + \rho)^t}$$

(23)

$$\text{PVC}_{\text{ex}}(C_{0\ldots T}; \rho, \bar{g}) = T \sum_{t=0}^{T} \frac{C_t}{(1 + \rho + \bar{g})^t}$$

(24)

We are now interested which reduction in consumption at $t = 0$ is ‘equivalent’ (in terms of these two SWFs) to a given reduction at $t = T$? For DU equivalence, we determine $a_0$ (depending on $a_T$, $T$, and $\rho$) as follows:

$$\text{DU}_{\text{DICE}}(C(\cdot; g, a_0, 0); \rho) = \text{DU}_{\text{DICE}}(C(\cdot; g, a_T, T); \rho)$$

$$\sum_{t=0}^{T} \frac{\ln C(t; g, a_0, 0)}{(1 + \rho)^t} \iff \sum_{t=0}^{T} \frac{\ln C(t; g, a_T, T)}{(1 + \rho)^t}$$
\[
\ln(1 - a_0) + \frac{\ln((1 + g)T)}{(1 + \rho)T} = \frac{\ln((1 + g)T \cdot (1 - a_T))}{(1 + \rho)T}
\]
\[
\ln(1 - a_0) = \frac{\ln(1 - a_T)}{(1 + \rho)T}
\]
\[
a_0 = 1 - (1 - a_T)((1 + \rho)^{-T})
\]

For PVC equivalence, we determine \(a_0\) (depending on \(a_T\), \(T\), \(g\), \(\rho\), and \(\tilde{g}\)) as follows:

\[
P_{\text{PVC}}(C(\cdot; g, a_0, 0); \rho, \tilde{g}) = P_{\text{PVC}}(C(\cdot; g, a_T, T); \rho, \tilde{g})
\]
\[
\sum_{t=0}^{T} C(t; g, a_0, 0) = \sum_{t=0}^{T} C(t; g, a_T, T)
\]
\[
(1 - a_0) + \frac{(1 + g)^T}{(1 + \rho + \tilde{g})^T} = 1 + \frac{(1 + g)^T \cdot (1 - a_T)}{(1 + \rho + \tilde{g})^T}
\]
\[
a_0 = a_T \cdot \left(\frac{1 + g}{1 + \rho + \tilde{g}}\right)^T
\]

The relationship between a reduction in future consumption, \(a_T\), and the equivalent reduction in current consumption, \(a_0\), for DU DICE and PVC \(\text{ex}\) is thus characterized as follows:

\[
a_{\text{DU}DICE}^{0}(a_T, T, \rho) = 1 - (1 - a_T)^{(1 + \rho)^{-T}}
\]
\[
a_{\text{PVC}}^{0}(a_T, T, g, \rho, \tilde{g}) = a_T \cdot \left(\frac{1 + g}{1 + \rho + \tilde{g}}\right)^T
\]

Fig. 3 depicts \(a_{\text{DU}DICE}^{0}(\cdot)\) and \(a_{\text{PVC}}^{0}(\cdot)\) across the whole range of \(a_T\) for three different points in time: \(T = 1\), \(T = 10\), and \(T = 100\) (assuming \(\rho = \tilde{g} = g = 3\%/\text{yr}\)). We observe that \(a_{\text{DU}DICE}^{0}(\cdot)\) and \(a_{\text{PVC}}^{0}(\cdot)\) are very similar for small values of \(a_T\) and/or \(T\). However, \(a_{\text{PVC}}^{0}(\cdot)\) is linear in \(a_T\) whereas \(a_{\text{DU}DICE}^{0}(\cdot)\) is non-linear, with \(\lim_{a_T \to 1} a_{\text{DU}DICE}^{0}(\cdot) = 1\).

Fig. 3 also shows for which combinations of \(a_0\) and \(a_T\) each of the two welfare metrics prefers the ‘late-loss path’ \(C(\cdot; g, a_T, T)\) over the ‘early-loss path’ \(C(\cdot; g, a_0, 0)\). The late-loss path is preferred by a welfare metric if an \((a_T, a_0)\) pair is above the corresponding curve; the early-loss path is preferred if the pair is below the curve. For instance, the black point lying between the lines marked ‘DU, T=10’ and ‘PVC, T=10’ indicates that a 80\% reduction of consumption at \(t = 10\) is preferred over a 65\% reduction at \(t = 0\) according to PVC \(\text{ex}\) whereas the reverse is true for DU DICE.

DU DICE and PVC \(\text{ex}\) produce identical rankings for the constant-growth consumption paths considered in Sect. 3.1. The present example shows, however, that these two SWFs may produce inconsistent rankings if the consumption paths involve significant welfare deviations at different points in time, even under the simplifying assumptions of constant population and discount rates, We suppose that this inconsistency has minor implications for the integrated assessment of climate change, compared to the other inconsistencies discussed here.
4.3 Expected consumption vs. expected utility

PVC and DU aggregate differently across possible states of the world in a probabilistic analyses. Table 2 provides a simple example for this inconsistency. The four left columns show consumption \( C \) and logarithmic utility \( U(C) = \ln C \) for two equally likely states of the world (SOW 1 and 2) and for two different policies (A and B). Policy A is associated with higher consumption (and utility) under SOW 1 whereas policy B leads to higher consumption (and utility) under SOW 2. The three right columns show three aggregated welfare measures \( E \) denotes the expected value). We find that policy A has higher expected consumption but policy B has higher expected utility. The right-most column shows the certainty equivalent \( C^* \) of the two policies, which was calculated such that \( U(C^*) = E(U(C)) \). A certainty equivalent is the certain result that would make an individual indifferent between it and the uncertain outcome.

The intuitive reason for the inconsistent policy rankings produced by expected logarithmic utility and expected consumption is that the former accounts for risk aversion whereas the latter does not. In this example, Policy A is more risky than policy B in the sense that the consumption difference between the two states of the world is much larger and, as a result, the certainty equivalent is considerably lower than expected consumption.
According to Arrow et al. (1996, p. 130), “Most economists believe that considerations of risk can be treated by converting outcomes into certainty equivalents, [...] and discounting these certainty equivalents”. While it is straightforward to compute certainty equivalents for individual time steps (as in Table 2), determination of their present value raises the question of the proper discounting scheme. Analogous to Sect. 4.1, it can be shown that the expected value of discounted logarithmic utility is inconsistent with the present value of a series of certainty equivalents for any discounting scheme. The importance of this inconsistency for a particular analysis requires careful examination by the analyst. Lind & Schuler (1998, p. 66) note that “we have never seen [a cost-benefit analysis] that has systematically converted costs or benefits to certainty equivalents”, arguing further that “In the case of most public projects of policies it is virtually impossible to do so”. In this context, it is important to note that sequential decision-making produces results that are superior to those of ‘classical’ cost-benefit analysis using certainty equivalents when there is the possibility of obtaining new information over time (Dixit & Pindyck, 1994).

5 Implications for climate policy analysis

In this section, we will apply the findings from Sect. 3 and 4 to the design of climate policy analyses based on the welfare maximization paradigm. Sect. 5.1 identifies four recommendations for the application of SWFs, and Sect. 5.2 reviews the use of SWFs in two recent analyses.

We note again that the uncritical application of welfare economics in the climate change context has been criticized for various reasons not addressed in this paper (see Sect. 1 for selected references). Our aim here is to help preventing the introduction of additional inconsistencies that may be caused by choosing flawed SWFs or by combining SWFs inappropriately.

5.1 Recommendations for the application of social welfare functions

The application of SWFs in optimizing climate policy analyses should obey the following recommendations in order to avoid the inconsistencies identified above:

1. Welfare metrics should be chosen based on the (supposed) preferences of target decision-makers; this choice should be made explicit.

2. Welfare metrics expressed in monetary terms need to implement a consistent discounting scheme.

3. The same welfare metric and discounting scheme should ideally be used in all optimizations and to report the relative ‘desirability’ of alternative policies. If different welfare metrics are combined in an analysis, the analyst needs to demonstrate that the inconsistencies between these metrics do not affect the conclusions of the analysis.

4. Social welfare cannot be objectively compared across different population scenarios.

What do these recommendations mean for the choice of SWFs for climate policy analysis according to the welfare maximization paradigm? If an analyst assumes risk-neutral decision-makers,
PVCex is an appropriate SWF. However, economists find very little empirical support for risk-neutral behaviour in individuals (Arrow et al., 1996). If an analyst believes that the degree of risk aversion and other preferences of target decision-makers are adequately reflected by the Bernoullian utility function, DU\textsubscript{DICE} is an appropriate SWF. This SWF is indeed very commonly used in economic models of climate change (DeCanio, 2003, Table 2.4), and it can be easily modified to accommodate different degrees of risk aversion. A potential disadvantage associated with non-monetary SWFs, such as DU\textsubscript{DICE}, is that they are expressed in arbitrary utility units. As a result, “Any economist doing this work will obviously feel a strong urge to discount the difference in the consumption streams to a present value” (Lind & Schuler, 1998, p. 80). Following this “strong urge”, some analysts have attempted to convert utility differences into monetary costs, defined as the difference in present value between alternative policies (see Sect. 5.2). However, the combination of different welfare metrics in an analysis is likely to introduce inconsistencies because different welfare metrics aggregate differently across components of economic output (Sect. 4.1), across time (Sect. 4.2), across possible states of the world (Sect. 4.3), and across regions and population groups (not discussed in this paper). Analysts who nevertheless combine different welfare metrics (e.g., by maximizing expected utility and reporting the present value of the certainty equivalents of consumption over time) need to demonstrate convincingly that the inconsistencies between these metrics do not affect the policy conclusions of the analysis. In the absence of such a demonstration, the analysis must be regarded as potentially inconsistent.

5.2 Application of social welfare functions in recent climate policy analyses

In this section, we review the use of welfare metrics in two recent applications of DICE that have not followed the recommendations presented in Sect. 5.1.

Yohe et al. (2004) describes a hedging analysis that aims to identify the optimum short-term policy under uncertainty about climate change and the long-term stabilization target. This uncertainty is characterized by several ‘policy cases’, which are characterized by a specific value for the climate sensitivity and an upper bound on the greenhouse gas (GHG) concentration level. Each policy case is assigned a probability based on an empirical probability density function (PDF) for climate sensitivity, assuming further that all considered GHG stabilization levels are equally likely. It is further assumed that the ‘true’ policy case will be revealed in 2035. A modified version of DICE-99 is used to determine the ‘optimal’ decision strategy for each policy case by maximizing DU\textsubscript{DICE} for different initial levels of the carbon tax (until 2035) and without such a constraint. For each of those utility-maximizing strategies, the discounted gross world product (GWP) is calculated according to PVO\textsubscript{Yohe}. The “discounted adjustment costs” for each policy case and initial carbon tax level are then defined as the difference in discounted GWP between the utility-optimal strategies with and without prescribing the initial carbon tax level. Finally, the “optimal” initial carbon tax level is determined by minimizing the expected discounted adjustment costs for each tax level, considering the probability of the various policy cases.

We argue that the use of welfare metrics in Yohe et al. (2004) involves several inconsistencies, with important implications for the results presented. Our first argument recalls the findings from Sect. 3.1, which shows that PVO\textsubscript{Yohe} is internally inconsistent and underestimates the welfare differences between alternative policies by about two orders of magnitude compared to...
PVO_{ex}, which uses a more consistent discounting scheme. Hence, the analysis presented in Yohe et al. (2004) violates the second rule for a consistent use of welfare metrics identified in Sect. 5.1. Fig. 4, which reproduces two diagrams from Yohe et al. (2004), shows the relevance of this problem for practical climate policy analysis. The discounted GWP difference between a 450 ppm CO\textsubscript{2} concentration target and a 900 ppm target in this figure is only 0.015\% (left panel), which is about two orders of magnitude smaller than the cost estimates from most other studies (Metz et al., 2001). Furthermore, variation in expected discounted GWP across all considered tax levels is a mere 0.0004\% (right panel). We argue that the internal inconsistencies associated with PVO_{Yohe} are the main reason for the surprisingly low variation in discounted GWP depicted in Fig. 4. Our second argument recalls the findings from Sect. 4.1 that DU_{DICE} and PVO_{ex} may produce inconsistent policy rankings. In Yohe et al. (2004), DU_{DICE} is initially maximized but PVO_{Yohe} is later used as the basis for selecting the ‘optimal’ policy. In violation of the third rule identified in Sect. 5.1, there is no discussion whether these two SWFs produce similar rankings for the policies considered in that analysis, or what the potential implications of the inconsistent rankings could be. Our third argument recalls the findings from Sect. 4.3 that expected consumption and expected logarithmic utility may produce inconsistent policy rankings due to different degrees of risk aversion. The same arguments hold in relation to Yohe et al. (2004), where expected output is maximized within a limited set of policy strategies that were initially determined by utility maximization in a deterministic context. We note that the analysis in Yohe et al. (2004) also violates the first rule identified in Sect. 5.1 since the authors do not state clearly which SWF they believe to best represent the preference structure of target decision-makers.

What are the implications of these flaws for the policy conclusions reported in Yohe et al. (2004)? The study concludes that “An initial $10 tax policy is remarkably robust across the remaining possibilities”, noting further that it is “surprising that climate insurance over the near term can be so inexpensive and that an economically efficient near-term hedging policy can be so robust across a wide range of futures in comparison with doing nothing”. We have shown that the reported costs of policies depicted in Fig. 4 are incorrect, most likely by about two orders of magnitude. We further note that even if the values depicted in Fig. 4 were correct, the tiny GWP variation across different policies would hardly support such a strong conclusion. Determining the correct optimal carbon tax level (subject to the assumptions of this particular analysis) would require a rerun of the whole modelling exercise in accordance with the three
rules identified above. Since such a reanalysis is beyond the scope of this paper, we cannot say for sure whether the recommended carbon tax level is correct.

Fankhauser & Tol (2005) apply DICE-94 to compare indirect climate impacts under different assumptions. This analysis defines several modifications to the production function of DICE-94, determines the optimal decision strategy for each model variant by maximizing the standard discounted utility function of DICE-94, \( DU_{\text{DICE}} \), and presents the time paths and growth rates of undiscounted GDP (corresponding to \( PVO_{\text{ex}} \) for \( \theta = \rho = 0 \)) for these decision strategies. Given the findings from Sect. 4.1 that \( DU_{\text{DICE}} \) and \( PVO_{\text{ex}} \) may produce inconsistent policy rankings, we have to consider the results of Fankhauser & Tol (2005) as potentially flawed. We also note that the endogenous determination of population growth suggested but not performed in Fankhauser & Tol (2005) would violate the last rule identified in Sect. 5.1 and introduce another inconsistency to the analysis (see Sect. 3.2).

## 6 Conclusions

Social welfare functions (SWFs) are applied in welfare-maximizing climate-economy models to compare alternative policies, either qualitatively or quantitatively. This paper presents a review of the main SWFs that have been applied in this context.

We show that the inappropriate implementation of growth discounting has resulted in various SWFs that are logically inconsistent and/or that produce counterintuitive results. We also find a close link between the index number problem and applying monetary SWFs across different consumption paths, which implies that there is no single ‘correct’ method for determining and comparing the present value of alternative climate policies. To our knowledge, this link has not been discussed in the pertinent literature so far. The various calculation methods applied by different analysts in the past may lead to cost estimates of achieving given climate policy targets that differ by several orders of magnitude. We further show that the various internally consistent SWFs are generally not interchangeable since they aggregate differently across time, population groups, states of the world, and components of economic output. From these theoretical findings, we draw four recommendations for the consistent use of SWFs in climate policy analyses according to the welfare maximization paradigm. A review of the application of SWFs in two recent analyses with the DICE model that do not follow these recommendations suggests that several key results are artifacts caused by the inappropriate choice of SWFs.

The inconsistencies identified in this paper are not only of theoretical interest. Each of them can strongly affect the policy recommendations drawn from a particular analysis. Therefore, our findings indicate that the specification and combination of SWFs in climate policy analyses require considerably more caution than has been exercised in the past. We note again that in addition to the logical inconsistencies discussed in this paper, there are many other conceptual, empirical, and philosophical problems associated with the identification of ‘optimal’ climate policy analysis by welfare maximization.
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References


A Different versions of the Ramsey growth discounting rule

Calculations of the costs of climate policies in integrated assessment models are usually based on the present value of economic output or consumption. In the ‘classical’ approach to time discounting (see, e.g., Tol, 1999), future discount factors are based on the ‘Ramsey rule’ for optimal saving (Ramsey, 1928). The ‘Ramsey growth discounting rule’ is typically stated as

\[ r_t = \rho_t + \theta \cdot g_t \]  
(27)

(see Sect. 2 for an explanation of the parameters). This formulation of the Ramsey rule is, however, only an approximation of the exact solution of the Ramsey model (see, e.g., DeCanio, 2003, Section 3.3.1):

\[ r = (1 + \rho) \cdot (1 + g) - 1 \]  
(28)

\[ \approx (1 + \rho) \cdot (1 + \theta \cdot g) - 1 \]

\[ = \rho + \theta \cdot g + \rho \cdot \theta \cdot g \]  
(29)

\[ \approx \rho + \theta \cdot g \]  
(30)

Eq. 28 gives the exact solution under the assumption that all parameters are constant over time. Eq. 28 and Eq. 29 are identical for \( \theta = 1 \), the standard assumption in integrated assessment models of climate change (see DeCanio, 2003, Table 2.4). Eq. 29 and Eq. 30 are identical for \( \rho = 0 \), a value that is only occasionally assumed in integrated assessments of climate change (for an example, see Yohe et al., 2004) even though it was originally recommended by Ramsey (1928). While the differences between Eq. 29 and Eq. 30 are generally negligible, they are relevant in the context of this paper where they explain the difference between the two welfare metrics PVC\(_{\text{DICE}}\) (based on Eq. 29) and PVC\(_{\text{end}}\) (based on Eq. 30).