Determination of the Distance to the Crab Nebula

MATTHEW J. BESTER & MATTEO J. PARIS

By comparing two photographic images of the Crab Nebula taken 52 years apart, we have calculated the angular velocity of expansion for 50 filamentary features of the nebular shell. We have also estimated the maximum linear velocity of the expansion in the direction perpendicular to the plane of the sky by analyzing the emission spectrum of the nebula. Combining these measurements indicates that the Crab is approximately 5,600 light years away. The factor contributing most to the uncertainty in this result is our lack of understanding of the three-dimensional shape of the ellipsoidal shell. In addition, if we assume that the nebula has been expanding at a constant rate, our angular velocity calculations date the supernova event to the year A. D. 1160, more than 100 years later than the accepted date of 1054. In order to more accurately determine the distance to the Crab Nebula, this discrepancy must also be explained.

Introduction

On July 4, 1054, a Chinese peasant and amateur astronomer may have looked up from his or her rice field to witness a spectacular stellar event. A bright object appeared suddenly and continued to shine its inexplicable light in the terrestrial sky for almost two years. The remnant of this explosion is known today as the Crab Nebula, so named by Lord Rosse in 1848.[1] How is it that we know that the nebula we see today corresponds to the supernova recorded almost a millennium ago? The Crab's age was estimated by Edwin Hubble in 1928, and later by others, using a method comparing two direct images of the Crab taken a number of years apart. Since the angular expansion is actually visible even over the span of only a decade, it is a simple matter of dividing the angular distance by the angular velocity to obtain an estimate of the Crab's age: about 900 years. A search through ancient Chinese and Native American Indian records convinced Hubble and others that a spectacular event described in the year 1054 matched the Crab Nebula's age and position correctly. In this project, in addition to repeating Hubble's measurements with better data, we also obtain the linear speed of the nebula's expansion along our line of sight by examining the Crab's spectrum. Using the linear and angular rates of expansion, we could then calculate the Crab's distance from the Earth.

The Crab Nebula, also known as M1, is an expanding spheroid of debris left over from the violent death of an especially massive star (Figure 1). It was the first such remnant found in our galaxy. The original star was most likely an O or B type star with ten or more solar masses. After completing its red giant phase of stellar evolution it reached the iron nuclear threshold, where energy must be absorbed in order to fuse the iron fuel rather than liberated as it is in reactions involving lighter elements. The star's only available source for this energy was gravitational collapse. The tremendous amount of heat generated by the collapse disintegrated the iron atoms into helium, resulting in a pressure decrease in the core. The core was then unable to support the large star, and another massive collapse occurred, this time taking less than a second. This final collapse caused electrons and protons to collide and create neutrons in the extremely hot core. As equilibrium was restored in the new neutron core, the collapsing outer layers of heavy elements produced during the star's later evolution bounced off the core and sent out a massive shock wave, sending the surrounding material into space at very high speeds. Remaining at the center of this powerful explosion was a neutron star whose magnetic fields and high rate of rotation continuously spur synchrotron radiation into space. Like a lighthouse, its radio emissions reach Earth every 0.033 seconds, making it one of the fastest pulsars recorded. In turn, this radiation heats up the bright network of knots and forms them into clumps, giving the filaments the crablike shape portrayed in Figure 1. In summary, we begin this project with the assumption that the filamentary nebular material visible today lies in the expanding shell, with the pulsar as the center of the expansion.

The Angular Velocity

We were very fortunate to have at our disposal two excellent images of the Crab, taken 52 years apart. The first was taken by Walter Baade in late 1941 on the Mount Wilson 100-inch telescope. It is one of a batch of pictures taken by Baade at this time; another appears in Owen Gingerich's *Sky and Telescope* article.[1] Professor Gingerich kindly provided us with a number of photographic prints obtained from several different sources, and we chose one to use based on its good condition and superb clarity. The print measures 19 by 24 cm, with a scale of (very roughly) 4 arc seconds per millimeter. The major and minor axes of the Nebula span

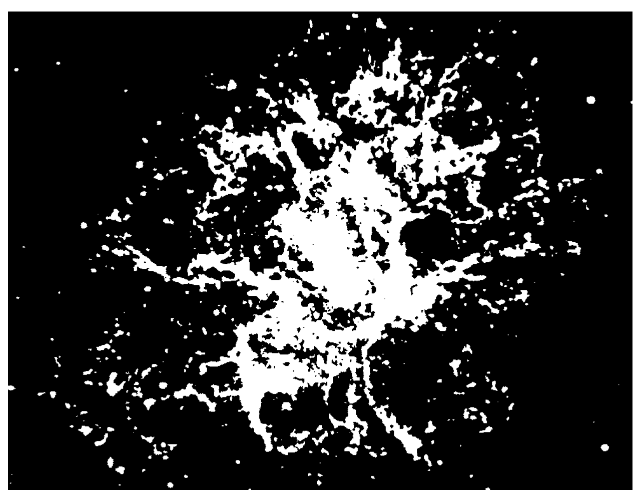

Figure 1. The Crab Nebula.

MATTHEW J. BESTER is a junior at Harvard College concentrating in Music and is planning to attend graduate school in musicology. He maintains an active interest in astronomy, and his recent work will be appearing in the journal *Astronomy & Astrophysics.* MATTEO J. PARIS is a recent graduate of Harvard College who is currently studying mathematics in graduate school at UCLA. Together they carried out the research described in this article in the spring of 1994 in the course *The Astronomical Perspective* under the supervision of Prof. David W. Latham of the Harvard-Smithsonian Center for Astrophysics.

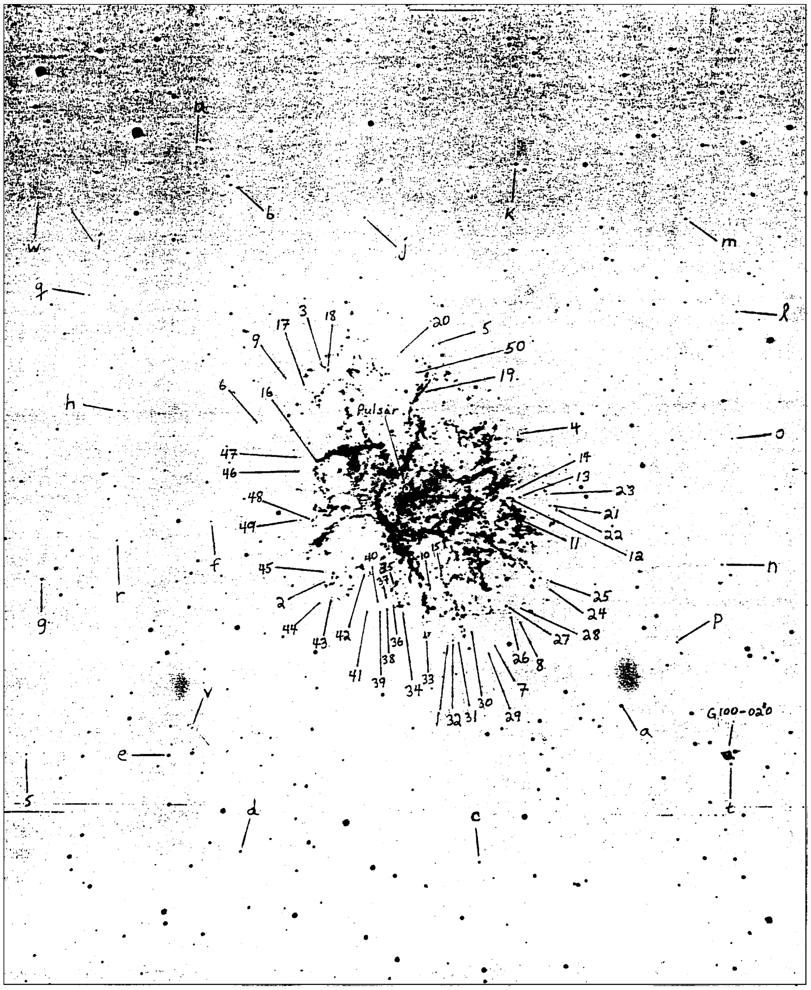

Figure 2. Stars are labeled a-w, knots are labeled 1-50. Some filamentary detail was lost in the reproduction process.

about 9 and 6 cm respectively, or roughly 6 and 4 minutes of arc. Measurements were taken using a clear millimeter ruler, to an accuracy of ±0.2 mm. We chose small, distinct features for all of our measurements; a typical one was about 0.4 mm in diameter. To give an idea of the scale of the measurements involved, a typical knot at the outer edge of the expanding shell moved about 2 mm during the half century between the times our images were taken. The distance a filament moves (equivalently, its angular velocity) is proportional to its distance from the center of expansion. Therefore, we preferred to choose filamentary features on the outer edge of the image, thereby increasing the distances measured and consequently decreasing the effect of measurement error.

Our second image was taken by Ian Dell'Antonio on February 11, 1994, on the 1.2 meter telescope at the Whipple Observatory in Arizona. The image was recorded with the modern method, using a CCD (Charge Coupled Device). A CCD is a microchip that contains an array (for example, 1024 by 1024) of light-sensitive pixels. Images that are focused onto the CCD are stored in a computer file for later viewing, refinement, and analysis. Obviously, CCD images have some great advantages over conventional photographs. First, because the image is computerized, measuring distances between features on the image is easier and slightly more accurate. Second, distortions inherent in photographic prints due to shrinking of the fibers during drying, an issue relevant to our other print, are nonexistent. Third, the CCD image saves significantly more visual information than a photograph. The first time we loaded the image onto a computer terminal, an uninspiring field of stars appeared on the screen with no nebula in sight. But as we adjusted the brightness scale, the Crab gradually materialized, detail by detail, until it blazed prominently in the middle of the previously empty picture. The wide range of visual information is quite helpful in locating features precisely.

We used IRAF (Image Reduction and Analysis Facility), the most common software package in use among astronomers, to perform two standard refinements on our CCD image of the Crab. This process removes artifacts from the image that were introduced by the instruments. Along with the file containing the image of the Crab, the observer provided us with a number of "bias" and "flat-field" files. Each bias file contains a different image of the readings of the CCD in complete darkness. This can be thought of as the background of the CCD. The bias is subtracted from the target image. To reduce the noise contributed by bias subtraction, we averaged two bias fields together before the subtraction. Next we made use of the flat-field images, which are images of a brightly lit, featureless white wall. This data encodes the variations in sensitivity of each pixel of the CCD. The target image is divided by the flat-field so that the brightness of the image does not encode the imperfections of the CCD. We averaged seven flat-field files together before the division.

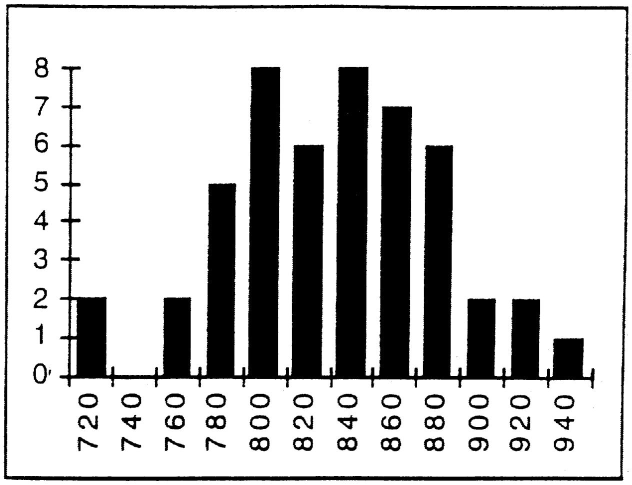

Figure 3. Distribution of the 50 age estimates.

Since the two images were taken at different times with different equipment and on different media, we first needed to establish a scaling factor between them. Fortunately, as a general rule background stars move very little, usually less than one arc second per century. Over 52 years this corresponds to a motion of only 0.13 mm on the 1941 image, which is actually smaller than our measurement error of ±0.2 mm. Thus we could establish a scale by comparing distances between fixed stars on each image, and a large sample size should have compensated for slight motions of any particular star. Having stated this assumption we should also mention a coincidence that contradicts it. In the course of scanning the star field meticulously for suitable small, distinct stars to use as measuring points, we noticed that one star had moved substantially: nearly 2 mm on the 1941 image. Using the Lowell Observatory Proper Motion Survey, Professor David Latham assisted us in identifying the star as G100-020 (G is for Giclas), one of only a few thousand rapidly moving stars that have been identified. G100-020 is rated at 13 arc seconds per century in the survey, which agrees nicely with our measurements of its motion.

We chose 8 pairs of stars, distributed roughly in a circle around the center of the nebula, so that each pair spanned a diameter of the circle of average length 150 mm on the 1941 image (Figure 2). We then divided each distance from the 1994 image by its corresponding distance on the 1941 image (Table 1). The resulting 8 ratios were not tightly distributed about the mean of the ratios. The largest differed by 3% from the smallest, which translates into a discrepancy of 3 mm for every 100 mm on the 1941 image; the standard deviation was 1.1% of the mean. Given that the background stars are supposed to remain fixed and that measurement error can account for a deviation of no more than ±0.2 mm over the 100 mm, these results clearly indicated that something was wrong.

Stars	1994 (pix)	1941 (mm)	Ratio
a b	888.40	151.7	5.856
c j	910.00	154.1	5.905
d k	1026.62	174.4	5.887
e m	1018.43	175.6	5.800
g l	1014.20	175.9	5.766
f o	712.57	124.3	5.733
h n	845.70	147.1	5.749
i p	1025.87	176.7	5.806
		average	5.813
		sigma	0.061

Table 1. First attempt to determine scale between images.

In fact, upon examining the data more closely, it became apparent that the variation in the figures was closely correlated to the orientation of the star pairs. The pairs oriented parallel to the 24 cm dimesion of the 1941 image (call this direction vertical) gave high values, while the pairs oriented horizontally gave low values. We hypothesized that the 1941 photographic paper had shrunk by about 3% in the horizontal direction, producing the poor scaling data. Indeed, Professor Gingerich had initially alerted us to this very phenomena when he gave us the prints. We tested the hypothesis by measuring 5 pairs of stars oriented vertically, and 5 pairs oriented horizontally. The results were quite convincing: the standard deviations dropped to 0.10% in the horizontal direction and 0.27% in the vertical direction, which are on the order of the measurement error. Our conclusion was that in order to convert measurements from one image to the other, we had to scale the horizontal and vertical components separately.

Next, we measured the distances from the pulsar to 50 knots on the periphery of the nebula (Figure 2). Since the measurements on the computerized 1994 image were obtained in coordinates, scaling the horizontal and vertical directions differently was not a problem. The scale on the 1994 image was determined to be uniformly 0.648 arc seconds per pixel (no distortions). Thus, we were able to find the angular velocity for each knot in arc seconds per year. Note that the angular velocities will be very different depending on where in the nebular shell the knot lies. Those with the greatest angular velocity will be the ones with no velocity component towards or away from Earth; these are the knots on the outer edge of our image of the Crab. Table 2 shows the angular velocity and the estimated date of convergence of each knot to the pulsar assuming a constant angular velocity.

The results were again satisfying. The average predicted age for the nebula was 833 years with a standard deviation of only 50 years, slightly smaller than the measurement error of ±80 years. Our distribution appears Gaussian and peaks exactly at the mean, suggesting that there were no major systematic errors involved in our measurements (Figure 3). For comparison, we computed ages using a single conversion scale between images. While the average predicted age was almost identical (830 years), the standard deviation was 140 years, and distribution was closer to being flat than Gaussian. Our standard deviation and mean values are good to two significant figures following standard practice in astrophysical methodology, as well as for the purposes of this comparison.

Much to our surprise, these calculations predicted a convergence of the nebular shell to the pulsar in the year 1160, more than 100 years later than 1054. If we insist upon the 1054 date for the supernova event, we are forced to conclude that the knots have been accelerating outwards. This is a very difficult conclusion to explain. If there is to be

Filament	Adjusted Scale (pix/mm)	Distance to pulsar 1994 (pix)	Distance to pulsar 1941, scaled	Angular Velocity (arcsec/year)	Convergence age (years)
1	5.869	237.27	223.60	0.170	907
2	5.833	185.94	173.83	0.150	802
3	5.835	222.00	208.30	0.170	846
4	5.800	184.71	174.00	0.133	901
5	5.891	211.11	198.52	0.156	876
6	5.777	219.51	206.25	0.165	865
7	5.844	261.02	245.47	0.193	877
8	5.834	260.41	243.87	0.205	823
9	5.823	253.50	238.16	0.190	863
10	5.874	148.75	138.04	0.133	725
11	5.777	151.34	142.10	0.115	856
12	5.749	152.60	143.16	0.117	845
13	5.742	162.65	153.32	0.116	911
14	5.736	150.00	140.54	0.117	828
15	5.858	140.27	130.63	0.120	760
16	5.774	149.11	139.72	0.116	830
17	5.817	207.57	193.71	0.172	782
18	5.835	213.30	198.98	0.178	778
19	5.885	143.93	133.59	0.128	727
20	5.883	240.56	224.71	0.197	793
21	5.749	207.74	195.45	0.152	884
22	5.751	218.16	205.33	0.159	888
23	5.735	201.30	189.27	0.149	874
24	5.805	262.51	245.55	0.210	809
25	5.802	245.18	230.33	0.184	863
26	5.829	233.78	223.24	0.180	855
27	5.828	226.33	212.14	0.177	831
28	5.824	236.07	221.31	0.183	836
29	5.852	275.71	257.47	0.226	790
30	5.853	233.10	217.71	0.191	792
31	5.864	255.96	239.24	0.207	800
32	5.866	236.38	221.73	0.182	843
33	5.873	223.88	209.09	0.184	791
34	5.908	186.32	176.05	0.127	948
35	5.897	171.53	161.58	0.123	901
36	5.892	182.46	170.26	0.151	782
37	5.889	184.83	173.15	0.145	827
38	5.890	190.58	179.05	0.143	864
39	5.884	192.91	180.05	0.160	783
40	5.883	187.53	175.90	0.144	842
41	5.868	188.39	175.44	0.161	760
42	5.858	144.50	134.73	0.121	773
43	5.844	196.57	184.08	0.155	822
44	5.828	229.57	214.47	0.187	794
45	5.824	180.40	169.47	0.136	862
46	5.751	164.37	154.14	0.127	839
47	5.765	179.04	168.34	0.133	874
48	5.786	147.27	138.87	0.104	916
49	5.782	164.35	153.80	0.131	814
50	5.895	183.95	172.12	0.147	812
				average	833
				sigma	49.5

Table 2. Angular velocity and convergence age for 50 filamentary features in the nebular shell.

Stars	1994(pix)	1941 (mm)	Ratio
q l	876.61	153.1	5.726
i m	837.05	145.8	5.741
h o	832.80	145.3	5.732
b n	822.70	143.4	5.737
s t	959.50	167.2	5.739
		average	5.735
		sigma	0.006

Table 3. Horizontal scaling data.

Stars	1994(pix)	1941 (mm)	Ratio
t l	628.65	106.6	5.897
c k	971.81	164.5	5.908
d b	924.96	156.2	5.922
v u	826.73	139.4	5.931
s w	771.50	130.9	5.894
		average	5.910
		sigma	0.016

Table 4. Vertical scaling data.

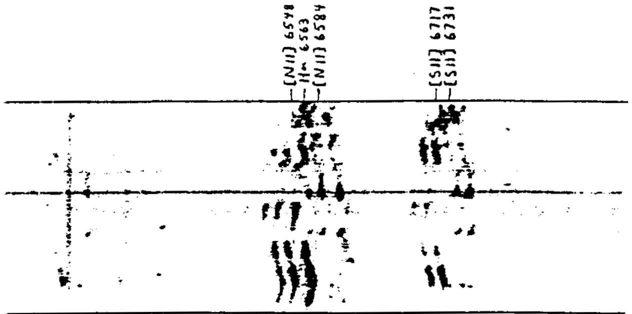

Figure 4. Necklace-shaped emission line spectrum of the Crab Nebula.

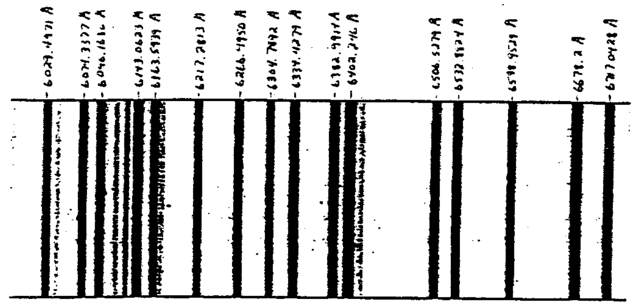

Figure 5. Comparison spectrum in the Hα region.

any change in angular velocity, we would expect the knots to be decelerating as they collide with interstellar material.

One possible explanation is that the electromagnetic radiation from the pulsar is so strong that it gives the knots a boost as it collides into them from behind. Brian Schmidt asserts that this effect becomes minimal within two days after the explosion (personal communication). Another possibility is that the features we measured in the 1994 image were not identical to those in the 1941 image due to the use of a different filter. However, Virginia Trimble obtained a similar result in a 1968 paper when she used several plates made with the same filter.[2] Trimble calculated the convergence to the year 1140. The fact that our calculated date is 20 years later than Trimble's, although promising for the acceleration hypothesis, is not statistically significant. Thus, although we cannot now explain the acceleration of the filaments, we have no reason to doubt the accuracy of the angular velocity calculations.

The Linear Velocity

Next we turned to finding the linear velocity of the expanding shell. This value can be found by examining the spectrum of the Nebula. Compared to the straight emission lines one would expect to see in examining the spectrum of a star or other stationary radiation source, the spectrum of an expanding nebular cloud looks quite different. Due to the Doppler shift caused by the relative motion of a radiation source towards or away from an observer, the emission lines of the nebula appear necklace-shaped (Figure 4).

By finding the maximum shift between the redshifted and blueshifted regions and plugging this into the Doppler formula,

$$\Delta\lambda/\lambda = v/c$$

we can find the relative velocity between the two branches. By simply dividing this number in two we find the linear velocity, or radial velocity, of the knots moving away from the pulsar.

The sources of these nebular emission lines are the intricate knots of the nebula. An interesting fact about these knots is revealed by looking closely at the necklace shape of the emission lines. No lines are detected between the two strands, indicating that all of the visible knots lie on the outer edge of the nebula either moving toward or away from Earth. No knots lie in between; only the synchrotron radiation continally emitted from the pulsar exists there.

The first step towards determining the linear velocity was to acquire a recent and accurate spectrum of the Crab. The spectrum we used was taken by Graydon Hazenberg using the FAST spectrograph on the Tillinghast telescope at the Whipple Observatory in Arizona on February 16, 1994.[3] After obtaining and analyzing the CCD image, we located two areas in the spectrum that had prominent emission lines and showed obvious Doppler shifts. Each emission line is characteristic of a particular element or ionized element present in the Nebula. We chose the region around the Hα lines and the area around the [O III] lines and made hard copies of each in order to make measurements. Before proceeding, however, we needed to obtain a wavelength scale for each spectra. To do so we further needed the comparison spectra, or the spectra by the FAST of a glowing thorium and argon lamp, for each of the two areas (Figure 5).

Once we had the comparison spectra, we referred to the FAST Manual[3] to identify the wavelengths in angstroms (Å) of the emission lines observed. We then measured the distance in millimeters of each emission line from a single line, and computed the average, i.e., the scale of per millimeter.

In the Hα region, the average slope was 4.01, while it was 3.99 in the [O III] region. To demonstrate the consistency in the results, the slopes of the Hα region were plotted using the graphing program "Mongo," which yielded the extraordinarily straight line shown in Figure 6.

After consulting a catalog of galaxy emission lines available over the Internet, we identified five emission lines; the Hα line, the two [N II] lines that flank it, and the two [O III]

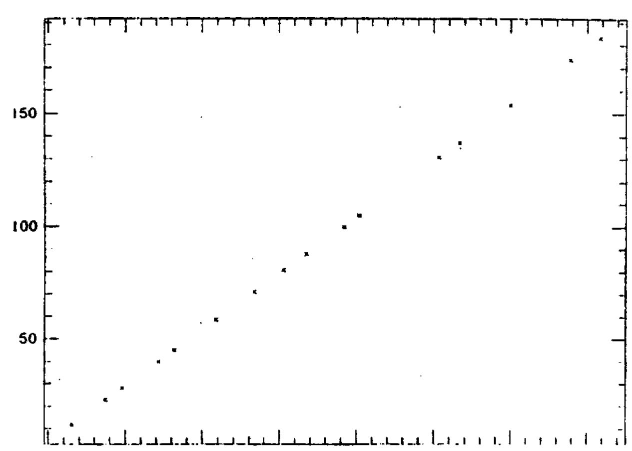

Figure 6. Scaling data in the Hα region.

Emission Line (Å)	Separation (mm)	Scale (Å/mm)	Separation (Å)	Rel. vel. (km/s)
[NII] 6548.06	14.3	4.01	57.343	2620
Hα 6562.82	14.2	4.01	56.942	2590
[NII] 6583.57	14.4	4.01	57.744	2620
[OIII] 5006.84	10.9	3.99	43.491	2600
[OIII] 4958.91	10.8	3.99	43.092	2600
			average	2606
			sigma	13

Table 5. Maximum relative velocities of approaching and receding knots.

lines at the other end of the spectrum. We found the Doppler shift for each emission line by measuring the maximum distance (in millimeter) between the two shifted lines and converting this figure into angstroms, being careful to use the correct scale for the region. The Doppler formula was then applied to give a linear velocity for the expanding shell (Table 5). For instance, the maximum separation in the Hα line was 14.2 mm, corresponding to 56.942 Å. By dividing this by the actual wavelength from the catalog of emission lines, 6562.817 Å, then multiplying this result by the speed of light, we obtain the relative velocity between the knots approaching and receding from Earth. This relative velocity works out to be about 2,590 km per second. The average velocity from the five examples was 2,606 ±13 km per second. The several outliers, including our Hα example, are well within the measurement error of ±43 km per second. By dividing by two we obtain a linear velocity of the knots moving outwards from the pulsar of 1,300 km per second.

Distance to the Crab

If we consider the linear motion of the filaments to be along a sphere, then the following relationship between angular velocity μ and linear velocity v is evident:

$$\mu/360^{o} = v/(2\pi d)$$

It follows that the distance d from the observer to the knot is given by the formula:

$$d = v/\mu$$

Of course, we have a problem in that we only know the maximum linear velocity v = 1,300 km/sec along the line of sight, i.e., for a knot coming straight towards Earth or going directly away from us. We do not know the length of the ellipsoid's axis along the line of sight. Suppose it is equal to the length of the major axis of the ellipse in the plane of the sky. Table 2 indicates a maximum angular velocity of μ = 0.21 arcsec/yr in the direction of the major axis, giving a distance of 4,270 light years (after adjusting for units). On the other hand, if we assume the axis perpendicular to us is as short as the minor axis of the ellipse, Table II tells us to use μ = 0.13 arcsec/yr, giving a distance of 6,900 light years. Finally if we simply use the average of the computed angular velocities, μ = 0.16 arcsec/yr, we obtain a distance of 5,610 light years.

Conclusion

To summarize, our best estimate of the distance to the Crab Nebula is 5,600 light years, although we can say with certainty only that it lies between 4,000 and 7,000 light years. Further study of the linear velocities of individual filaments would be very useful in determining the three-dimensional shape of the nebula. This might also lead to an explanation for the apparent acceleration of the knots. Recent studies using the refurbished Hubble Space Telescope have already made valuable insights about the filamentary structure of the Nebula and its interaction with the radiation inside which may lead to a better understanding of what is occurring there. Of course, the remote possibility exists that the observed nebula is not actually the remnant of the 1054 event. There remain numerous shadowy secrets lurking within the Crab Nebula, waiting to be exposed by future sleuthing and slewing astronomers. On the other hand, if SETI is a success, we can always ask somebody more intelligent for the answers.

Acknowledgments

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