1. Classical Mechanics - 20%

(such as kinematics, Newton's laws, work and energy, oscillatory motion, rotational motion about a fixed axis, dynamics of systems of particles, central forces and celestial mechanics, three-dimensional particle dynamics, Lagrangian and Hamiltonian formalism, noninertial reference frames, elementary topics in fluid dynamics)

- v in terms of x in uniform acceleration.

$$v^2 = v_0^2 + 2a(x - x_0)$$

- Lagrangian. Let T =kinetic energy, U =potential energy.

$$L = L(q, \dot{q}, t) = T - U$$

- Euler-Lagrange equations of motion.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$$

- Action. Pick the path with the correct endpoints that minimizes

$$S = \int_{t_0}^{t_1} L(q(t), \dot{q}(t), t)dt$$

- Conjugate momentum.

$$p = \frac{\partial L}{\partial \dot{q}}$$

- Hamiltonian.

$$H = p\dot{q} - L = T + U$$

- Hamiltonian equations of motion.

$$\dot{q} = \frac{\partial H}{\partial p}$$
$$\dot{p} = -\frac{\partial H}{\partial q}$$

- Bernoulli's Equation for fluid flow. (Assume incompressible, nonviscous, laminar flow; this equation holds along flowlines, or if the flow is irrotational, it holds everywhere.) Take g as positive.

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

2. E&M - 18%

(such as electrostatics, currents and DC circuits, magnetic fields in free space, Lorentz force, induction, Maxwell's equations and their applications, electromagnetic waves, AC circuits, magnetic and electric fields in matter)

- ε and μ (names, $\vec{E}$ and $\vec{D}$, $\vec{H}$ and $\vec{B}$, χ_e, χ_m, n, c).

$$\varepsilon = \text{“permittivity”}$$
$$\mu = \text{“permeability”}$$
$$\vec{D} = \varepsilon \vec{E}$$
$$\vec{H} = \frac{1}{\mu}\vec{B}$$
$$\varepsilon = \varepsilon_0(1+\chi_e) \quad (\chi_e = \text{“electric susceptibility”})$$
$$\mu = \mu_0(1+\chi_m) \quad (\chi_m = \text{“magnetic susceptibility”})$$
$$n = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \frac{c}{c_{\text{eff}}}$$
$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

- Typical values of ε, μ.
  - Permittivity: $\varepsilon > \varepsilon_0$ almost always, with $\chi_e > 0$.
  - Diamagnetics: $\mu < \mu_0$, $\chi_m < 0$. No unpaired electrons. Field is reduced by Lenz's law acting on electron orbitals.
  - Paramagnetics: $\mu > \mu_0$, $\chi_m > 0$. Has some unpaired electrons, which align with and increase the magnetic field.
  - Ferromagnetics: $\mu >> \mu_0$ (although of course not linear). Unpaired electrons, domains, and such and such.

  How to remember dia versus para: “In a diamagnet, the magnetic field dies.”

- Maxwell's equations.

$$\text{Gauss's Law:} \quad \vec{\nabla}\cdot\vec{D} = \rho_{\text{free}}; \quad \int_{\partial V} \vec{D}\cdot\vec{n}dA = Q_{\text{enclosed}}$$
$$\text{No monopoles:} \quad \vec{\nabla}\cdot\vec{B} = 0; \quad \int_{\partial V} \vec{B}\cdot\vec{n}dA = 0$$
$$\text{Faraday's Law:} \quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}; \quad \int_{\partial S} \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\vec{B}_{\text{flux through}}$$
$$\text{Ampère's Law (modified):} \quad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}; \quad \int_{\partial S} \vec{H}\cdot d\vec{\ell} = I_{\text{through}} + \frac{d}{dt}D_{\text{flux through}}$$

- Induced voltage from Faraday's Law. $\Phi =$flux of magnetic field through coil, $N =$number of turns. Sign is determined by Lenz's Law.

$$V = N\frac{d\Phi}{dt}$$

- Coulomb's Law.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

- Biot-Savart Law.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{I}d\vec{L} \times \hat{r}}{r^2}$$

- Magnetic field on axis of a circle of current.

$$\mu = \frac{\mu_0 I}{2}\frac{r^2}{(r^2 + z^2)^{3/2}}$$

$$\mu = \frac{\mu_0 I}{2r}, \qquad \text{at center of loop}$$

- Magnetic field from infinite straight wire. Direction of $\vec{B}$ from right-hand rule.

$$B = \frac{\mu_0}{2\pi}\frac{I}{R}$$

- Force on a wire.

$$F = I\, d\vec{\ell} \times \vec{B}$$

- Boundary conditions for $\vec{E}$, $\vec{D}$, $\vec{H}$, $\vec{B}$ in macroscopic media. In absence of surface currents and surface charge, the normal components of $\vec{D}$ and $\vec{B}$ are continuous, and the tangential components of $\vec{E}$ and $\vec{H}$ are continuous.

- Capacitor – capacitance and energy.

$$Q = CV$$

$$U = \frac{1}{2}CV^2$$

- Capacitance of a parallel-plate capacitor.

$$C = \frac{\varepsilon A}{d}$$

- Inductor – inductance and energy.

$$V = -L\frac{dI}{dt}$$

$$U = \frac{1}{2}LI^2$$

- Inductance of a solenoid. A =cross sectional area, l =length, N =number of turns.

$$L = \frac{\mu N^2 A}{l}$$

Why N^2? If you double the coils, you double the flux and you double the response per unit flux. Another memory trick: the equation has almost the same form as the capacitance of a parallel-plate capacitor.

- Impedance. $(V_0 e^{i\omega t}) = (I_0 e^{i\omega t})Z$, where

$$\text{Resistor:} \quad Z = R$$

$$\text{Capacitor:} \quad Z = \frac{1}{i\omega C}$$

$$\text{Inductor:} \quad Z = i\omega L$$

How to remember: at $\omega = 0$ (DC), capacitor has infinite resistance, and inductor has 0 resistance.

3. Optics and Wave Phenomena - 9%

(such as wave properties, superposition, interference, diffraction, geometrical optics, polarization, Doppler effect)

- Phase velocity versus group velocity.

$$v_{phase} = \frac{\omega}{k}$$

$$v_{group} = \frac{d\omega}{dk}$$

- Doppler shift (for waves in a medium). Top signs when moving towards each other.

$$\omega = \omega_0 \frac{1 \pm \frac{v_o}{c}}{1 \mp \frac{v_s}{c}}$$

How to remember: If source moves towards observer at c, infinite frequency; if observer moves away from source at c, zero frequency (picture the waves).

- Thin lens formula. Figure out signs by ray-tracing.

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

- Rayleigh Criterion. D is lens diameter, θ is angular resolution.

$$\sin\theta \approx 1.22\frac{\lambda}{D}$$

- Focal length of lens/mirror.

$$\text{MIRROR:} \quad f = \frac{R}{2}$$

$$\text{LENS:} \quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(sign convention: R_1, R_2 both positive for a converging lens.) (replace n by n/n' if the surrounding medium isn't a vaccuum.)

4. Thermodynamics and Statistical Mechanics - 10%

(such as the laws of thermodynamics, thermodynamic processes, equations of state, ideal gases, kinetic theory, ensembles, statistical concepts and calculation of thermodynamic quantities, thermal expansion and heat transfer)

- Carnot cycle. Start in top-left of P-V diagram and go around clockwise. Isothermal expansion at T_H (heat enters), adiabatic expansion, isothermal compression at T_C (heat exits), adiabatic compression. Efficiency is $(T_H - T_C)/T_H$. When volume is expanding, system does work $\int PdV$, so clockwise orientation of loop implies that the system does net positive work.

- Boltzmann distribution.

$$Z = \sum_s e^{-E_s/kT}$$

$$P(s) = e^{-E_s/kT}/Z$$

- Gibbs distribution.

$$\mathcal{Z} = \sum_s e^{(N_s\mu - E_s)/kT}$$

$$P(s) = e^{(N_s\mu - E_s)/kT}/\mathcal{Z}$$

- Relation between entropy and heat. Heat added (reversably) to the system is $\int TdS$.

5. Quantum Mechanics - 12%

(such as fundamental concepts, solutions of the Schrödinger equation (including square wells, harmonic oscillators, and hydrogenic atoms), spin, angular momentum, wave function symmetry, elementary perturbation theory)

- Time-Independent Perturbation Theory: 2nd order for energy, 1st order for wave function. Notation: If $H = H_0 + \lambda\Delta H$, then $E_n = E_n^{(0)} + \lambda E_n^{(1)} + \cdots$, and $|n\rangle = |n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \cdots$.

$$E_n^{(1)} = \langle n^{(0)}|\Delta H|n^{(0)}\rangle$$

$$|n^{(1)}\rangle = \sum_{k\neq n} \frac{\langle k^{(0)}|\Delta H|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}}|k^{(0)}\rangle$$

$$E_n^{(2)} = \sum_{k\neq n} \frac{|\langle k^{(0)}|\Delta H|n^{(0)}\rangle|^2}{E_n^{(0)} - E_k^{(0)}}$$

(In the latter two equations, deal with degeneracies by picking correct basis (so ΔH is diagonal in the degenerate subspace), then summing just over those k nondegenerate with n.)

- Heisenberg Uncertainty Principle.

$$(\sigma_A)(\sigma_B) \geq \left|\frac{1}{2}\langle[A, B]\rangle\right|$$

$$(\sigma_p)(\sigma_x) \geq \frac{\hbar}{2}$$

- de Broglie Wavelength.

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$$

- Pauli spin matrices. $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$, where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

6. Atomic Physics - 10%

(such as properties of electrons, Bohr model, energy quantization, atomic structure, atomic spectra, selection rules, black-body radiation, x-rays, atoms in electric and magnetic fields)

- Electron quantum numbers.

- $n = 1, 2, 3, \ldots$ – principle quantum number, controls radial wave function.
- $\ell = 0, 1, \ldots, n-1$ – orbital quantum number, controls θ wave function. If $\vec{L} = \vec{r} \times \vec{p}$, then $L^2\psi = \hbar^2 \ell(\ell+1)\psi$.
- $m_\ell = -\ell, -\ell+1, \ldots, \ell-1, \ell$ – magnetic quantum number, controls ϕ wave function. Also, $L_z \psi = \hbar m_\ell \psi$.
- $m_s = \pm(1/2)$ – electron spin quantum number. Eigenvalue of S_z, where $\vec{S}$ is $\hbar/2$ times the pauli matrices.
- $j = |\ell \pm (1/2)|$ – total angular momentum quantum number. If $\vec{J} = \vec{L} + \vec{S}$, then $J^2|\psi\rangle = \hbar^2 j(j+1)|\psi\rangle$.
- $m_j = -j, -j+1, \ldots, j-1, j$ – z-component of total angular momentum. Note that $m_j = m_\ell + m_s$, and $J_z|\psi\rangle = \hbar m_j |\psi\rangle$.

- Electric dipole transition. Selection rules are as follows:

$$\Delta\ell = \pm 1 \quad (\neq 0)$$

$$\Delta m_\ell = 0, \pm 1$$

$$\Delta j = 0, \pm 1$$

$$\Delta m_s = 0$$

- Sources of line-splitting in the spectrum.
  - Zeeman Effect: When you apply a uniform external magnetic field, each transition energy $E_{n_1\ell_1 \to n_2\ell_2}$ is split into three equally-spaced lines, due to whether m_ℓ increases by one, decreases by one, or stays the same in the transition.
  - Anomalous Zeeman Effect: In Zeeman effect, the contribution of electron spin to total angular momentum means that it isn't always three lines and they are not always equally spaced.
  - Stark Effect: When you apply a uniform electric field, it induces a dipole moment and interacts with it, and that effect depends on $|m_j|$. So if j is an integer, splits (asymmetrically) into $j+1$ levels, and if j is a half integer, splits (asymmetrically) into $j + 1/2$ levels.

- Bohr model. Assume classical orbiting with angular momentum a multiple of $\hbar$. Z is an integer representing nuclear charge. m_{red} is (reduced) electron mass $= m_1 m_2/(m_1 + m_2)$. n is orbit.

$$\text{Radius:} \quad a_n = \frac{4\pi\varepsilon_0\hbar^2}{m_{\text{red}} e^2 Z} n^2 \approx (0.529\text{Å}) \left(\frac{m_{\text{elec}}}{m_{\text{red}}} \cdot \frac{n^2}{Z} \right)$$

$$\text{Energy:} \quad E_n = \frac{Z^2 m_{\text{red}} e^4}{8\varepsilon_0^2 h^2} \left(\frac{-1}{n^2} \right) \approx (-13.6\text{eV}) \left(\frac{Z^2}{n^2} \cdot \frac{m_{\text{red}}}{m_{\text{elec}}} \right)$$

- X-ray spectrum from electrons fired at atoms. "Auger transition" is when incoming particle knocks out inner-shell electron, and vacancy gets filled by outer-shell electron, creating a spike in the spectrum. "Bremsstrahlung" is the continuous spectrum of light released from the deceleration of electrons. Put them together to get the full spectrum (a continuous spectrum with a few spikes on top).

7. Special Relativity - 6%

(such as introductory concepts, time dilation, length contraction, simultaneity, energy and momentum, four-vectors and Lorentz transformation, velocity addition)

- Lorentz factor.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1$$

$$\gamma \approx 1 + \frac{1}{2}\frac{v^2}{c^2}, \qquad \frac{1}{\gamma} \approx 1 - \frac{1}{2}\frac{v^2}{c^2}$$

- Lorentz transformations. Let $\beta = \frac{v}{c}$.

$$ct' = \gamma\,(ct - \beta x)$$

$$x' = \gamma\,(x - \beta(ct))$$

$$y' = y$$

$$z' = z$$

- Relativistic addition of velocities. According to a guy moving at speed v in the x-direction, a ball is thrown with velocity u. Rest frame velocity of the ball is u', where:

$$u'_x = \frac{u_x + v}{1 + u_x v/c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 + u_x v/c^2)}$$

$$u'_z = \frac{u_z}{\gamma(1 + u_x v/c^2)}$$

(Derivation: plug in $x = u_x t$, Lorentz transform, compute x'/t'.)

- Length contraction.

$$L' = \frac{L_{proper}}{\gamma}$$

- Time dilation.

$$T' = T_{proper}\gamma$$

- Doppler shift (for light).

$$\omega = \omega_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$$

- Relativistic momentum.

$$p = \frac{mv}{\gamma}$$

8. Laboratory Methods - 6%

(such as data and error analysis, electronics, instrumentation, radiation detection, counting statistics, interaction of charged particles with matter, lasers and optical interferometers, dimensional analysis, fundamental applications of probability and statistics)

- Accuracy versus Precision. Accuracy: Close to reality. Precision: Repeatable. How to remember: "Precise can be repeated thrice, accurate is on track."

9. Specialized Topics - 9%

Nuclear and Particle physics (e.g., nuclear properties, radioactive decay, fission and fusion, reactions, fundamental properties of elementary particles), Condensed Matter (e.g., crystal structure, x-ray diffraction, thermal properties, electron theory of metals, semiconductors, superconductors), Miscellaneous (e.g., astrophysics, mathematical methods, computer applications)

- Conservation of baryon/lepton number.
  - Baryon number. Each quark has baryon number 1/3.
  - Lepton number. Three types: electron number, muon number, tau number. Each separately is preserved.
- Binding energy. A positive quantity, such that

$$(\text{binding energy})/c^2 = (\text{total mass of constituent nucleons}) - (\text{mass of nucleus})$$