GRE Review: Lab Methods

Officially, this section contains: “data and error analysis, electronics, instrumentation, radiation detection, counting statistics, interaction of charged particles with matter, lasers and optical interferometers, dimensional analysis, fundamental applications of probability and statistics”

After looking through the practice tests, the key things to review are how to calculate errors, how lasers work, how to read data charts or instrumentation output, and how to calculate I, R, and V given a circuit diagram (i.e. Kirchoff’s Law, series vs. parallel, capacitance, etc.).

A good textbook to skim over for the basics is Giancoli “Physics for Scientists and Engineers: Volume II”.

(And I noticed a few practice problems involving the Michelson Interferometer – check out http://scienceworld.wolfram.com/physics/MichelsonInterferometer.html.

B. CAPACITORS

The Capacitance of two oppositely charged conductors in a uniform dielectric medium is

\[ C = \frac{Q}{V_0} \]

Units: Farads = coul/\(V\)

where \(Q\) = the total charge in either conductor
\(V_0\) = the potential difference between the two conductors.

EXAMPLE:

Capacitance of the parallel-plate capacitor:

\[ E = \frac{\rho_s}{\varepsilon} a_z \]

\(\varepsilon\) is the permittivity of the homogeneous dielectric

\[ D = \rho_s a_z \]

On lower plate:

\[ D_n = D_z = \rho_s \]

\(D_n\) is the normal value of D.

On upper plate:

\[ D_n = -D_z \]

\(V_0\) = The potential difference

\[ V_0 = \frac{\varepsilon_s}{\varepsilon} \]

\[ C = \frac{Q}{V_0} = \frac{\varepsilon_s}{d} \]
\[ Q = \rho_s S \text{ and } V_0 = \frac{\rho_s d}{\epsilon} \]

considering conductor planes of area \( S \) are of linear dimensions much greater than \( d \).

Total energy stored in the capacitor:

\[
W_E = \frac{1}{2} \int_{\text{vol}} \varepsilon E^2 dV = \frac{1}{2} \int_0^S \int_0^{\rho_s} \varepsilon_2 dz \, ds
\]

\[
W_E = \frac{1}{2} CV^2 = \frac{1}{2} QV_0 = \frac{1}{2} \frac{Q^2}{C}
\]

Multiple dielectric capacitors

A parallel-plate capacitor containing two dielectrics with the dielectric interface parallel to the conducting plates;

\[
C = \frac{1}{\left( \frac{1}{C_1} \right) + \left( \frac{1}{C_2} \right)}
\]

where \( C_1 = \frac{\varepsilon_1 S}{d_1} \)

\( C_2 = \frac{\varepsilon_2 S}{d_2} \)

\( V_0 = \) A potential difference between the plates

\( = E_1 d_1 + E_2 d_2 \)

\( E_1 = \frac{V_0}{d_1} + \left( \frac{\varepsilon_1}{\varepsilon_2} \right) d_2 \)

\( \rho_{s1} = \) The surface charge density = \( D_1 = \varepsilon_1 E_1 \)

\( = \left( \frac{d_1}{\varepsilon_1} \right) + \left( \frac{d_2}{\varepsilon_2} \right) = D_2 \)

\( C = \frac{Q}{V_0} = \frac{\rho_{s} S}{V_0} = \frac{1}{\left( \frac{d_1}{\varepsilon_1 S} \right) + \left( \frac{d_2}{\varepsilon_2 S} \right)} = \frac{1}{\left( \frac{1}{C_1} \right) + \left( \frac{1}{C_2} \right)} \)

C. CURRENT AND RESISTANCE

DEFINITIONS

Current: \( i = \frac{dq}{dt} \) amperes
where \( i \) = Electric Current
\( q \) = Net Charge
\( t \) = Time

Current Density and Current:

\[
j = \frac{i}{A} \text{ Amperes/m}^2
\]

where \( j \) = Current Density
\( i \) = Current
\( A \) = Cross-sectional Area

**Mean Drift Speed:**

\[
v_D = \frac{j}{ne}
\]

where \( v_D \) = Mean Drift Speed
\( j \) = Current Density
\( n \) = Number of atoms per unit volume.

**Resistance:**

\[
R = \frac{V}{i} \text{ Ohms (Ω)}
\]

where \( R \) = Resistance
\( V \) = Potential Difference
\( i \) = Current

**Resistivity:**

\[
\rho = \frac{E}{j} \text{ Ohm - meters (Ω m)}
\]

where \( \rho \) = Resistivity
\( E \) = Electric Field
\( j \) = Current Density

**Power:**

\[
P = VI = I^2R = \frac{V^2}{R} \text{ Watts (w)}
\]

where \( P \) = Power
\( I \) = Current
\( V \) = Potential Difference
\( R \) = Resistance

**D. CIRCUITS**

**Electromotive Force, EMF(ε)**

\[
ε = \frac{dw}{dq}
\]
where $\varepsilon = \text{Electromotive Force}$  
$w = \text{Work done on Charge}$  
$q = \text{Electric Charge}$

**Current in a Simple Circuit**

\[
i = \frac{\varepsilon}{R}
\]

where $i = \text{Current}$  
$\varepsilon = \text{Electromotive Force}$  
$R = \text{Resistance}$

Resistances:

\[
R_{\text{Total}} = (R_1 + R_2 + R_3) \, \Omega \, (\text{in series})
\]

\[
\frac{1}{R_{\text{Total}}} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \, (\text{in parallel})
\]

**The Loop Theorem**

\[
\Delta V_1 + \Delta V_2 + \Delta V_3 \ldots \ldots = 0
\]

For a complete circuit loop

**EXAMPLE**

Simple circuit with resistor
\[
V_{ab} = \varepsilon - iR = +ir
\]
\[
\varepsilon - iR - ir = 0
\]

Then
NOTE: If a resistor is traversed in the direction of the current, the voltage change is represented as a voltage drop, \(-iR\). A change in voltage while traversing the \textit{EMF} (or battery) in the direction of the \textit{EMF} is a voltage rise \(+\varepsilon\).

Circuit With Several Loops

\[ \sum_{n} i_n = 0 \]

EXAMPLE

\[ i_1 + i_2 + i_3 = 0 \]

Multiloop circuit

RC CIRCUITS (RESISTORS AND CAPACITORS)
RC charging and discharging

Differential Equations

\[ \varepsilon = R \frac{dq}{dt} + \frac{q}{C} \] (Charging)
\[ O = R \frac{dq}{dt} + \frac{q}{C} \] (Discharging)

Charge in the Capacitor

\[ q = (C\varepsilon) \left(1 - e^{-\frac{t}{RC}}\right) \] (Charging)
\[ q = (C\varepsilon) e^{-\frac{t}{RC}} \] (Discharging)
Charge in the Resistor

\[
\begin{align*}
  i &= \left( \frac{\varepsilon}{R} \right) e^{-\frac{t}{RC}} \quad \text{(Charging)} \\
  i &= -\left( \frac{\varepsilon}{R} \right) e^{-\frac{t}{RC}} \quad \text{(Discharging)}
\end{align*}
\]

where \( e = 2.71828 \) (Exponential Constant)

KIRCHOFF’S CURRENT LAW

The algebraic sum of all currents entering a node equals the algebraic sum of all current leaving it.

\[ \sum_{n=1}^{N} i_n = 0 \]

KIRCHOFF’S VOLTAGE LAW (SAME AS LOOP THEOREM)

The algebraic sum of all voltages around a closed loop is zero.

THEVENIN’S THEOREM

In any linear network, it is possible to replace everything except the load resistor by an equivalent circuit containing only a single voltage source in series with a resistor \( (R_{th} \text{ Thevenin resistance}) \), where the response measured at the load resistor will not be affected.

Procedures to Find Thevenin Equivalent:

1) Solve for the open circuit voltage \( V_{oc} \) across the output terminals.
2) Place this voltage \( V_{oc} \) in series with the Thevenin resistance which is the resistance across the terminals found by setting all independent voltage and current sources to zero. (i.e., short circuits and open circuits, respectively.)

RLC CIRCUITS AND OSCILLATIONS

These oscillations are analogous to, and mathematically identical to, the case of mechanical harmonic motion in its various forms. (AC current is sinusoidal.)

SIMPLE RL AND RC CIRCUITS

Source Free RL Circuit

\[ i(t) \]

\[ \begin{array}{c}
\text{L} \\
V_L
\end{array} \]
**Properties:** Assume initially $i(0) = I_0$.

A) \[ v_R + v_L = RI + L \frac{di}{dt} = 0 \]

B) \[ i(t) = I_0 e^{-Rt/L} = I_0 e^{-t/\tau}, \quad \tau = \text{time constant} = \frac{L}{R} \]

C) Power dissipated in the resistor = \[ P_R = i^2R = I_0^2 Re^{-2\pi t/\tau}. \]

D) Total energy in terms of heat in the resistor = \[ W_R = \frac{1}{2} LI_0^2. \]

Source Free RC Circuit

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**Properties:** Assume initially $v(0) = V_0$

A) \[ C \frac{dv}{dt} + \frac{v}{R} = 0 \]

B) \[ v(t) = v(0)e^{t/RC} = V_0 e^{-t/RC}. \]

C) \[ \frac{1}{C} \int_{-\infty}^{t} i(\tau)d\tau + i(t)R = 0 \]

\[ i(t) = i(0)e^{\frac{t}{\tau}} \]

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**THE RLC CIRCUITS**

Parallel RLC Circuit (source free)

Circuit Diagram:

KCL equation for parallel RLC circuit:

\[ \frac{v}{R} + \frac{1}{L} \int_{t}^{\infty} vdt - i(t_0) + C \frac{dv}{dt} = 0; \]

and the corresponding linear, second-order homogeneous differential equation is

\[ C \frac{d^2v}{dt^2} + \frac{1}{T} \frac{dv}{dt} + \frac{v}{L} = 0 \]

General Solution:

\[ v = A_1 e^{S_1t} + A_2 e^{S_2t}; \]
where

\[ S_{1,2} = \frac{-1}{2RC} \pm \sqrt{\left( \frac{1}{2RC} \right)^2 - \frac{1}{LC}} \]

or

\[ S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} ; \]

where \( \alpha = \) exponential damping coefficient never frequency \( = \frac{1}{2RC} \)
and \( \omega_0 = \) resonant frequency \( = \frac{1}{\sqrt{LC}} \)

COMPLETE RESPONSE OF RLC CIRCUIT

The general equation of a complete response of a second order system in terms of voltage for an RLC circuit is given by.

\[ v(t) = V_f + Ae^{S_1t} + Be^{S_2t} \]

(i.e., constant for DC excitation)

**NOTE:** \( A \) and \( B \) can be obtained by

1) Substituting \( v \) at \( t = 0^+ \)
2) Taking the derivative of the response, i.e.,

\[ \frac{dv}{dt} = 0 + S_1Ae^{S_1t} + S_2Be^{S_2t} \]

where \( \frac{dv}{dt} \) at \( t = 0^+ \) is known.

D. THE LASER

Light Amplification by Stimulated Emission of Radiation

**Properties**

1. The light is coherent. (Waves are in phase)
2. Light is nearly Monochromatic. (One wavelength)
3. Minimal divergence

Many atoms have excited energy levels which have relatively long life-times. (\( 10^{-3} \)s instead of \( 10^{-8} \) s). These levels are known as **metastable**.
Through a process known as population inversion, the majority of an assembly of atoms is brought to an excited state.

Population inversion can be accomplished through a process known as optical pumping, where atoms of a specific substance, such as ruby, are exposed to a given wavelength of light. This wavelength is enough to excite the ruby atoms just above metastable level. The atoms rapidly lose energy and fall to the metastable level.

Once population inversion has been obtained, induced emission can occur from photons dropping from an excited metastable state to ground state. The photons have a wavelength equal to the wavelength of photons produced by each individual atom. The radiated light waves will be exactly in phase with the incident waves, resulting in an enhanced beam of coherent light. Hence the familiar Laser effect.