Introduction

This guide is a synthesis of other documents on the website of the Society of Physics Students, my past lecture notes and summaries, and my experience doing the practice/real test papers. I took the test during the fall semester of the senior year and only had two days to fully devote myself to the preparation. I basically went over the materials in this guide and practiced three sample tests. It covers almost all topics in the test. If you fully master them, you should be able to get over 85 questions correct. I hope this document would prove helpful if you are under time constraint. However, it is advisable that you spend roughly a week’s time to work through all four available practice tests. You’re welcome to improve upon this version of the guide and to type up this document in Latex for online sharing.

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EXAM TIPS

* Pick moderate statements. Extreme statements are usually wrong.

* Use Taylor expansion to deal with certain extreme cases.
  - e.g. $h v < k T$, $e^{h v / k T} \approx 1 + h v / k T$

* When knowing $L^2$ value, be careful to calculate $L$ from $L^2 \propto (h / E)$, two solutions.

* Conservation of momentum (including angular momentum) should be checked before conservation of energy.

* Be careful about dimension of the problem, e.g. in 3D, radial use $P = \int |v|^2 \, dr = \int (v_r^2 + v_\theta^2 + v_z^2) \, dr$.

* Read underlined words carefully.

* Calculate $T^4$ carefully. It is 4th power!

* Don't think too hard, the questions are easy enough to be solved in 2min.

* Use method of elimination.

* Dimensional analysis is always useful.

* Usually order of magnitude calculation is good enough.

* Potential is a SCALAR, be careful.

* In general, $F = -\nabla \phi$ (potential energy) but in E&M, notice $\mathbf{V}$ stands for potential, not potential energy, so $\mathbf{F} = -\nabla \phi$.

* Usually it is convenient to set $h = \hbar = c = \ldots = 1$, but if this differs from choices, that's a signal that maybe we need to keep them.

* When you get stuck, take limits.

* If you haven't realized how important it is, I'll repeat: TAKE LIMITS.

* If some experimenter is involved in the question, it's usually a failed experiment (as GRE is usually set by theorists, according to yoshism.com.)
A worked example on velocity and acceleration in a curved path in a plane
\[ \vec{r} = \hat{r} \cos \theta + \hat{r} \sin \theta, \quad \dot{\vec{r}} = -\hat{r} \sin \theta + \hat{r} \cos \theta \]
\[ \vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dt} + \frac{dR}{dt} = \vec{v} + R \Omega \frac{d\vec{r}}{dt} \]
\[ \vec{a} = (R - \vec{w} \times \vec{v}) = (R - \vec{w} \times (R - \vec{w} \times \vec{v})) \]

The idea is skillfully use \( d(AB) = A dB + B dA \)
This applies to change of momentum as well.

- Firing rocket
\[ (v_f - V) \, dM + d(M \vec{v}) = 0 \]
\( M \) is rocket mass, \( V \) is speed, \( v_f \) is relative speed of the waste fired out.

- Bernoulli's equation
\[ P + \frac{1}{2} \rho v^2 + \rho g y = \text{const} \]
(conservation of energy)

- Torricelli's Theorem: The outlet speed is the free-fall speed.
For a barrel with water depth \( d \), an outlet at base has horizontal flow speed \[ v = \sqrt{2gd} \]

- Stokes' Law: viscosity drag = \( 6\pi \eta r^2 v \)
(Probably will not appear in GRE)

- Poiseuille's Law
\[ \Delta P = \frac{8 \eta Q L}{\pi r^4} \]
\( L \) is length of tube.
(again, too hard for GRE)
This describes viscous incompressible flow through a constant circular cross-section.

- Kepler's laws
1. (If you don't know this, wait a couple years before you take GRE)
2. \[ \frac{dA}{dt} = \frac{1}{2} r^2 \dot{\theta} = \text{constant} \]
3. \[ T = \frac{A}{2 \pi \dot{r}^2} = \frac{1}{2} \sqrt{\frac{m}{k}} R^2 \]
\( t \propto R^2 \)

- Eötvös = \( -2m \left( \vec{J} \times \vec{v} \right) \)
(unlikely)

- Diffusion: Fick's law
\[ \vec{J} = J_n \text{ (diffusion flux)} = -D \nabla \phi \]

- Frequency of a pendulum of arbitrary shape
\[ w = \sqrt{\frac{m g r}{I}} \]

- Hamiltonian formulation
\[ H = \frac{1}{2} \vec{p}^2 + V \]
\( \vec{p} = -\frac{\partial H}{\partial \vec{q}} \)
\( \dot{\vec{q}} = \frac{\partial H}{\partial \vec{p}} \)

- Circular orbits exist for almost all potentials. Stable non-circular orbits can occur for the simple harmonic potential and the inverse square law.
* Orbit questions, $V_{\text{eff}}(r) = V(r) + \frac{L^2}{2m^2}$
  $V(r) \propto \frac{1}{r}$ for gravitational potential
Total energy of an object $E = \frac{1}{2} m v^2 + V_{\text{eff}}$
$E < V_{\text{min}}$, a spiral orbit
$E = V_{\text{min}}$, a circular orbit
$V_{\text{min}} < E < 0$, ellipse
$E = 0$, parabolic
$E > 0$, hyperbolic
II ELECTROMAGNETISM

* Faraday's law of electrolysis (unlikely in GRE)
  (a) The mass liberated or charge passed through
  \[ m = \frac{\sigma A}{F} \]
  \[ A = \text{atomic weight} \]
  \[ V = \text{valence} \]
  \[ F = 9.65 \times 10^4 \text{ C/mol} \]
  (Faraday)

* Parallel plate capacitor
  \[ C = \varepsilon_0 \frac{A}{d} \quad \text{or} \quad \frac{C}{\varepsilon_0} \frac{A}{d} \quad \text{for dielectric} \]

* Spherical capacitor
  \[ C = \frac{4\pi \varepsilon_0 a b}{a - b} \]

* In charging a capacitor
  \[ q = q_0 (1 - e^{-t/RC}) \]
  Discharging
  \[ q = q_0 e^{-t/RC} \]

* Cyclotron/magnetic bending
  \[ R = \frac{m v}{qB} \] (easily derived)

* Torque experienced by a planar coil of N loops, with current I in each loop
  \[ T = NIAB \sin \theta, \quad \theta \text{ is angle between } B \text{ and line perpendicular to coil plane} \]
  \[ F = F_{n} \times B \]

* B-field of a long wire
  \[ B = \frac{\mu_0 I}{2\pi r} \]
  Center of a ring wire
  \[ \frac{\mu_0 I}{2\pi} \] (can generalize to arc)

* Center, long solenoid
  \[ B = \mu_0 n I \quad \text{n is turn density} \]

* Conductors do not transmit EM wave, thus E vector is reversed upon reflection, B vector is increased by a factor of 2 (by solving propagation of EM wave)

* \[ B = \mu_0 I = \mu_0 (H + N\lambda I) \]
  Diemagnetic \[ \Rightarrow \chi \text{ is very small & negative constant} \]
  Paramagnetic \[ \Rightarrow \chi \text{ is small & positive, inversely proportional to the absolute temp.} \]
  Ferromagnetic \[ \Rightarrow \chi \text{ is positive, can be greater than } 1 \]
  \[ \chi \text{ is no longer proportional to } H \]

* For solenoid and toroid
  \[ H = NI \quad n \text{ is no density} \]

* Self-inductance
  \[ L = \frac{\Phi_s}{I} \]
  L is in henries
  \[ L = \frac{\mu_0 N^2 A}{\ell} \]
  \[ \Phi_s = L I \text{ flux linkage} \]
  Primary inductance of solenoid
  \[ L = \frac{\mu_0 N^2 A}{\ell} \]

* Induced e.m.f.
  \[ |E_s| = N \left| \frac{\Delta \Phi}{\Delta t} \right| = N s \left| \frac{\Delta \phi}{\Delta t} \right| \]
Time constant for R-L circuit $t = \frac{1}{RC}$ for R-C is $t = RC$

Frequency for L-C circuit $w_0 = \frac{1}{\sqrt{LC}}$

$X_L = 2\pi f L$ inductive reactance $X_C = \frac{1}{2\pi f C}$ capacitive reactance

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ for series

$\frac{1}{Z} = \left[ \frac{1}{R^2} + \left( \frac{1}{X_L} - \frac{1}{X_C} \right) \right]^{\frac{1}{2}}$ for parallel

Current is maximized at resonance $X_L = XL = X_C = \frac{1}{LC}$

Larmor formula for radiation $P = \frac{M_0 q^2 a^2}{6\pi c} \times q \cdot a^2$

$q =$ charge $a =$ acceleration

Nuclear energy per unit area decreases as distance increases inverse square relation

Mean drift speed $\bar{V}_o = \frac{q n}{ne} = \frac{q}{e} n$ no. of atoms per volume $\bar{V}_o = \frac{q}{e} n$

Impedance $Z = \frac{1}{2\pi f C}$ of capacitor $Z = \frac{i}{2\pi f L}$ inductor

Magnetic field on axis of a circle of radius $r$ current $I$

$B = \frac{M_0 I}{2 \left( r^2 + 2r^2 \right)^{\frac{3}{2}}}$

Bremsstrahlung: electromagnetic radiation produced by the deceleration of a charged particle.

For incident wave reflecting off a plane, just set up a boundary value problem $\vec{E}_1^+ = -\vec{E}_1^-$ and remember Poynting vector $\dot{S} = \vec{E} \times \vec{B}$ points in the direction of propagation $\vec{E}_0$ reflected $= \vec{E}_0$

Lenz's law: The idea is the system responds in a way to restore or at least attempt to restore to original state, see 0117 Q.2

Impedance matching to maximize power transfer or to prevent terminal-end reflection $Z_{load} = Z_{source}$

$Z(L) + Z(C) = \frac{E}{I}$

Generator impedance $= R_g + jX_g$

Load impedance $= R_L + jX_L$

$Z = R + j(wL - \frac{1}{wC})$
\* Propagation vector $k \cdot \mathbf{E}(x, t) = \mathbf{E}_0 \mathbf{e}^{i(k \cdot \mathbf{x} - \omega t)}$.
\[ \frac{\partial}{\partial t} \mathbf{E}(x, t) = \frac{1}{c} \left( \mathbf{E}(x, t) \times \frac{\partial \mathbf{B}}{\partial x} \right) \]
\* No electric field in a constant potential enclosure implies constant potential inside.
\* Hall effect $R_H = \frac{1}{(p-n) e}$ for positive, $n$ for negative.
\* Can be used to test the nature of charge carrier.
\* Lorentz force (of course) $\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$.
\* $\nabla \cdot (\nabla \times \mathbf{H}) = 0$, $\nabla \times (\nabla \times \mathbf{E}) = 0$.
\* One usually have cyclone motion whenever the electric and magnetic fields are perpendicular.
\* Faraday's law $\mathbf{e}_{\text{m}} = \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$.
\* Visible spectrum.
\* Radio: $10^3$ buildings; Microwave: $10^{-2}$; Infrared: $10^{-5}$; Visible: $700$–$400$ nm; Ultraviolet: $10^{-8}$ molecules; X-ray: $10^{-12}$ Atoms; Gamma ray: $10^{-12}$ Nuclei.
\* $\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} + \varepsilon_0 \chi \varepsilon \mathbf{E} = \varepsilon_0 (1 + \chi \varepsilon) \mathbf{E} = \varepsilon \mathbf{E}$.
\* $\varepsilon_0 = 1 + \chi \varepsilon = \varepsilon_0$ (dielectric constant).
\* $\mathbf{D} = \varepsilon \mathbf{E}$ and $\mathbf{E}$ are bound.
\* $\mathbf{D} = -\nabla \chi \varepsilon$ not necessarily zero.
\* $\mathbf{B} = \mu_0 \mathbf{n} \mathbf{E}$, inside the solenoid, $n$ = density per length.
\* $\mathbf{B} = 0$ outside solenoid.
\* $\mathbf{B}(\mathbf{r}) = \int \frac{\mu_0 n m i \mathbf{e}}{2\pi r} \hat{\mathbf{n}}$, for points inside curid $\mathbf{r}$.
\* Toroidal $\mathbf{B}$.
\* Toroidal $\mathbf{B}$.
\* Force between two wires $F = \frac{\mu_0}{2\pi} \frac{I_i I_j}{d}$. Force per length.
\* $B = \mu_0 \frac{l}{2r} (\sin \theta_1 - \sin \theta_2)$.
\* Mutual inductance of two loops $M_{ij} = \frac{\mu_0}{4\pi} \int \int \frac{d i_1 \cdot d i_2}{d}$.
\* Radiation pressure $P = \frac{1}{c} \left( \frac{2\pi}{c} \right)$ for perfect reflector.
\* $\nabla \cdot \mathbf{B} = \mathbf{0}$, $\nabla \times \mathbf{H} = \mathbf{J}_Q + \frac{\partial \mathbf{D}}{\partial t}$, $\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}$.
\* Boundary conditions:
\* $\mathbf{E} + \mathbf{E} = \mathbf{E}$.
\* $\mathbf{B}_{x, 1} - \mathbf{B}_{x, 2} = 0$.
\* $\mathbf{B}_{y, 1} - \mathbf{B}_{y, 2} = 0$.
\* $\mathbf{B}_{z, 1} - \mathbf{B}_{z, 2} = 0$.
\* $\mathbf{E} \times \mathbf{H} = \mathbf{E} \times \mathbf{H}$. 

* Biot-Savart law: \( \vec{B}(r) = \frac{\mu_0}{4\pi} I \int \frac{\vec{d} \vec{r} \times \vec{I}}{r^2} \)

* B-field at center of a ring
  current: \( \vec{B} = \frac{\mu_0 I}{2\pi r} \)

* \( H = \frac{1}{\mu_0} \vec{B} - \vec{M} \quad \vec{J}_0 = \nabla \times \vec{M} \quad \vec{K}_0 = \vec{M} \times \hat{\mathbf{n}} \)

* \( \vec{B} = \mu_0 \vec{H} \quad \vec{M} = \mu_0 (1 + \chi m) \)

* Practically \( H \) is more important than \( D \), though they're equal theoretically.
**III OPTICS AND WAVE PHENOMENA**

* Speed of propagation for waves
  
  - Transverse on string \( v = \sqrt{\frac{F}{m}} \)
  
  - Longitudinal in liquid \( v = \sqrt{\frac{F}{\rho}} \) \( B \) is Bulk modulus
  
  - in solid \( v = \sqrt{\frac{F}{\rho}} \) \( Y \) is Young's modulus
  
  - in gases \( v = \sqrt{\frac{F}{\rho}} \)

* For open pipe, fundamental frequency \( f = \frac{V}{2L} \)
  
  - \( V \) is speed of sound. For closed pipe it is \( (2n+1)\frac{V}{4} = L \)
  
  - The idea is \( \gamma \cdot \frac{\lambda}{f} = V \) wavelength \( \cdot \) frequency = speed

* Speed of sound in air \[ c = \sqrt{\frac{RT}{M}} \] \( r \propto \sqrt{T} \)

* Resonant frequency of a rectangular drum \[ f_{mn} = \frac{V}{\pi} \sqrt{\left( \frac{L_m}{2L} \right)^2 + \left( \frac{L_n}{2L} \right)^2} \]

* Doppler Effect \[ f^\text{source} = f^\text{listener} \pm \frac{V_s}{V_s + V_r} f^\text{source} \]
  
  - \( V_s \) is source velocity w.r.t. medium
  
  - \( V_r \) is receiver velocity w.r.t. medium

  - In general \[ f^\text{receiver} = f^\text{source} \pm \frac{V_s}{V_s + V_r} f^\text{source} \]

  - The sign can be easily determined by examining if the frequency received is higher or lower.

  - Relativistic Doppler Effect see section VII

* Lens optics \( \frac{1}{f} = \frac{1}{R} + \frac{1}{R} = \frac{1}{f} \) sign convention

  - real image \( + \)
  
  - converging lens \( + \)
  
  - concave mirror \( - \)

  - Now if a real image is input and lies to the right of the lens, take it as \( - \) for left \( \rightarrow \) right process, or \( + \) for right \( \rightarrow \) left process

* Lens maker's equation \[ \frac{1}{f} = \frac{(n-1)}{(R_1 - R_2)} \]
  
  - \( R_1 \) positive \( \rightarrow \) convex
  
  - negative \( \rightarrow \) concave
  
  - \( R_2 \) positive \( \rightarrow \) concave
  
  - negative \( \rightarrow \) convex

* Young's double slit
  
  - \( d \sin \theta = m \lambda \) maxima
  
  - \( d \sin \theta = m \lambda \) for \( d \ll D \), \( \theta \) small

  - \( d \sin \theta = (m+\frac{1}{2}) \lambda \) minima

* Slab
  
  - \( \frac{2n_1}{n_1} = m + \frac{1}{2} \) max
  
  - \( \frac{2n_2}{n_2} = m + 1 \) min
Diffraction grating \( d \sin \theta = n \lambda \)

If incident at angle \( \theta \) 

\( d (\sin \theta + \sin \phi) = n \lambda \)

The overall result is interference pattern modulated by single slit diffraction envelope.

Intensity at interference 

\[ I = I_0 \frac{\sin^2 \left( \frac{\phi}{2} \right)}{\sin^2 \left( \frac{\phi}{2} \right)} \]

\( \phi = \frac{2\pi}{\lambda} d \sin \theta \)

minima occurs at \( \frac{n \phi}{2} = \pi, \ldots, n\pi \); \( \frac{n \phi}{2} \) not integer

maxima occurs at \( \frac{\phi}{2} = 0, \pi, 2\pi, \ldots \)

Single slit envelope

\[ I = I_0 \frac{\sin^2 \left( \frac{\phi}{2} \right)}{\left( \frac{\phi}{2} \right)^2} \]

\( \phi = \frac{2\pi}{\lambda} w \sin \theta \)

Overall

\[ I = I_0 \frac{\sin^2 \left( \frac{\phi}{2} \right)}{\left( \frac{\phi}{2} \right)^2} \frac{\sin^2 \left( \frac{\phi}{2} \right)}{\left( \frac{\phi}{2} \right)^2} \frac{\sin^2 \left( \frac{\phi}{2} \right)}{\left( \frac{\phi}{2} \right)^2} \]

\( \phi = \frac{2\pi}{\lambda} w \sin \theta \)

e.g., maxima will be missing when interference is max and single slit is min

Bragg's law of reflection

\( m \lambda = 2d \sin \theta \), careful \( \theta \) is glancing angle, not angle of incidence

Brewster's angle 

\[ \tan \theta = \frac{n_2}{n_1} \]

Diffraction again (more background info)

The light diffracted by a grating is found by summing the light diffracted from each of the elements, and is essentially a convolution of diffraction and interference patterns.

Fresnel diffraction (near-field)

Fraunhofer diffraction (far-field)

Diffraction-limited imaging

\( d = 1.22 \lambda / N \) 

\( N = \frac{\text{focal length}}{\text{diameter}} \) (call F number)

Angular resolution is \( \sin \theta = 1.22 \frac{\lambda}{D} \), \( D \) is lens aperture

Thin-film theory

\( \sin \theta \) film has higher refractive index

then phase change for reflection off front surface

no phase change for reflection off back surface

Constructive interference thickness \( t : 2t = (n+\frac{1}{2}) \lambda \)

Destructive interference \( 2t = n\lambda \)

The key idea for many questions is to scrutinize path difference, optical
* convex \( \rightarrow \) concave

- Objective \( \rightarrow \) eye piece
- Angular magnification = \( \frac{f_{\text{object}}}{f_{\text{eye}}} \)

- Magnifying power = max angular magnification = \( \frac{\text{image size with lens}}{\text{image size without lens}} \)

* Microscopy

- Magnifying power = \( \frac{\beta}{l} \)

* In Michelson interferometer a change of distance \( \frac{\lambda}{2} \) of the optical path between the mirrors generally results in a change of \( \lambda \) of optical path of light ray, thus potentially giving a cycle of bright \( \rightarrow \) dark \( \rightarrow \) bright fringes

* Mirror with curvature \( f \approx \frac{R}{2} \)

* Beats beat frequency is \( f = f_2 - f_1 \)

\[ \sin(2\pi f_1 t) + \sin(2\pi f_2 t) = 2 \cos(\pi f - \frac{f_1 + f_2}{2}) \sin \left( \frac{f_1 - f_2}{2} t \right) \]
Heat Transfer

Conduction: rate \( H = \frac{\Delta Q}{\Delta t} = -kA \frac{T_s - T_i}{L} \), \( \frac{dQ}{dt} = -kA \frac{dT}{dx} \), \( A \) is area, \( k \) is a constant.

Convection (probably not in GRE): \( H = \frac{\Delta Q}{\Delta t} = hACT_s (T_s - T_a) \)

\( T_s \) = surface temp., \( h \) = convective heat-transfer coefficient.
There are both natural & forced convections.

Radiation: Power \( = E\sigma A T^4 \), \( \sigma = 5.67 \times 10^{-8} \text{W/(m}^2\text{.K}^4) \) Stefan-Boltzmann constant.
\( E \) = emissivity, \( E \in [0, 1] \), Net loss = \( E\sigma A (T^4 - T_a^4) \)

Wien's displacement law: The absolute temperature of a blackbody and the peak wavelength of its radiation are inversely proportional (\( \lambda_m \cdot T = 2898 \mu \text{m.K} \) need to memorize)

\[ PV = nRT = N\frac{kT}{k} \]
\( N = \frac{2}{k} \) = no. of molecules

Kinetic Theory of gas

\( P = \frac{3}{8} kT \text{V}^\frac{2}{3} \text{m}^{-1} \), \( \text{V}_\text{rms} = \sqrt{\frac{3kT}{m}} \), \( \text{V}_{\text{max}} = \left( \frac{2kT}{m} \right)^{\frac{3}{2}} \)

Maxwell-Boltzmann distribution (less likely to be in GRE)

\( N(E) dE = \frac{2N}{\pi kT^3} \sqrt{E} e^{-\frac{E}{kT}} dE \)

\( N(E) dE = \frac{2N}{\pi kT^3} \sqrt{E} e^{-\frac{E}{kT}} dE \)

\( f(E)dE = \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} e^{-\frac{mV^2}{2kT}} dV \)

Mean free path of a gas molecule of radius \( b \)

\( \lambda = \frac{1}{4\pi b^2 \sqrt{N/N}} \)

van der Waals equation of state (unlikely to be in GRE)

\( (P + an^2/b^2) CV_{\text{bs}} = nRT \)

Adiabatic process \( PV^\gamma = \text{const} \)

For an ideal gas to expand adiabatically from \((P_1, V_1) \rightarrow (P_2, V_2)\), work done by the gas is \( W = \int P_{V_{1 \rightarrow 2}} (\text{derived from} \int P_{V_{1 \rightarrow 2}}) \)

The greatest possible thermal efficiency of an engine operating between two heat reservoirs is that of a Carnot engine, one that operates in the Carnot cycle.

Max efficiency is \( \eta = 1 - \frac{T_\text{cold}}{T_\text{hot}} \)

For the case of refrigeration \( \frac{Q}{W} = \frac{T_\text{cold}}{T_\text{hot}} \)

\( K = \frac{Q}{W} \) Carnot = \( \left( \frac{T_\text{hot}}{T_\text{cold}} - 1 \right)^{-1} \)

Carnot = adiabatic + isothermal \( dS = 0 \) (entropy constant)

Otto = adiabatic + isobaric \( \eta = 1 - \frac{T_d - T_a}{T_c - T_b} \)
* Dalton's Law \( P = P_1 + P_2 = (n_1 + n_2) \frac{RT_2}{V} \)
* The critical isotherm is the line that just touches the critical liquid-vapor region \( \frac{dP}{dT} = 0 \), \( \frac{d^2P}{dT^2} = 0 \), critical point equilibrium region where pressure and chemical potential for the two states of matter equal, usually a pressure constant region in \( P-V \) diagram.
* \( C_v = \frac{dE}{dT} = 3R \) in the Dulong-Petit law
* Laws of Thermodynamics (wiki)
  1st: If two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other.
  2nd: \( \Delta U = \Delta W \) heat flows from hot to cold
  3rd: \( T \to 0, S \to \text{constant minimum} \) 1968 Nobel Prize in Chemistry
  4th: Many versions, one is Onsager reciprocal relations
  5th: Your call.
* Partition function \( Z = \sum \frac{e^{-\beta E_i}}{Z} = \int dE \, w(E) \, e^{-\beta E} = \int dE \, e^{-\beta A(E)} \) Helmholtz free energy
  \( P(E_i) = \frac{e^{-\beta E_i}}{Z} \), entropy \( = k \ln w(E) = k \sum P(E_i) \ln P(E_i) \)
* Equipartition Thm: Classical canonical quantized dependence
  \( \Rightarrow \langle \Delta x^2 \rangle = \frac{1}{2} kT \) for each degree of freedom
  e.g. \( H_2 \) has 2 degrees of freedom
* Evolution energy \( dU = TdS - PdV \)
  Enthalpy \( H = U + PV \) \( dH = TdS + VdP \) isobaric
  Helmholtz \( F = U - TS \) \( dF = -SdT - PdV \) isothermal
  Gibbs Free energy \( G = U - TS + PV \) \( dG = -SdT + VdP \)
* \( C_v = \left( \frac{\partial U}{\partial T} \right)_v = T \left( \frac{\partial S}{\partial T} \right)_V \)
  \( C_p = \left( \frac{\partial U}{\partial T} \right)_p + P \left( \frac{\partial V}{\partial T} \right)_p = T \left( \frac{\partial S}{\partial T} \right)_p = \left( \frac{\partial H}{\partial T} \right)_p \)
\[ E = -\frac{2}{3} k \ln z \quad F = -k T \ln z \quad S = k \ln z + \frac{E}{T} \quad dS = \frac{P dV}{T} \]

\[ U = F + TS = F - T \left( \frac{\partial F}{\partial T} \right)_V = -T^2 \left( \frac{\partial^2 (\frac{E}{T})}{\partial T^2} \right)_V \left( \frac{E}{T} \right) \quad \text{Gibbs - Helmholtz equation} \]

* Availability of a system \( \Delta A = Q + P \Delta V - T \Delta S \)
  In natural change, \( A \) cannot increase.

* Ideal gas \( PV = nRT = n k_B T \)

* Diatomic gas \( U = \frac{5}{2} kT \)

* Maxwell relations: simple result based on multivar. calculus

* For ideal gas, in adiabatic process \( W = \Delta U = \frac{3}{2} n kT \)
  is the work done by gas in a cycle.

* Chemical potential \( \mu (T, V, N) = \left( \frac{\partial F}{\partial N} \right)_T, V \)
  at equilibrium \( \mu \) is uniform, \( \mu \) achieves minimum.

* \( P_{\text{boson}} \propto T^{5/2}, P_{\text{classical}} \propto T, P_{\text{phonon}} \propto T^{3/2} \)

  \( T_{\text{classical}} \gg T_{\text{boson}} \)

* A thermodynamic system in maximal probability state is stable.

* Both debye and Einstein assume \( JN \) independent harmonic oscillators for lattice. Einstein took a constant frequency.
Quantum Mechanics

1. Uncertainty Principle: \( \Delta x \Delta p \geq \frac{\hbar}{2} \)
2. Schrödinger equation: \( i \hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V \psi \) (1D case, can be extended)
4. De Broglie relation: \( \lambda = \frac{\hbar}{p} = \frac{\hbar c}{E} \) (for thermal wavelengths)
5. 1-dimensional problem has no degenerate states
6. Heisenberg's uncertainty: \( \Delta A \Delta B \geq \frac{1}{2} |\langle A, B \rangle| \) in particular \( \Delta x \Delta p \geq \frac{\hbar}{2} \)
7. Infinite square well: \( \psi_n = \frac{1}{\sqrt{2a}} \sin \frac{n\pi x}{a} \) \( E_n = \frac{n^2 \pi^2}{2a^2} \) \( n \geq 1 \) \( E_n \propto n^2 \)
8. Delta-function well: \( V = -\alpha \delta(x) \)
9. Only 1 bound state, many scattering states
10. \( \psi(x) = \frac{1}{\sqrt{I}} e^{-ax^2} \) \( J = -\frac{m^2 c^2}{2\hbar^2} \)
11. Shallow, narrow well, there are always at least one bound state
12. Selection rule: \( \Delta l = \pm 1 \) \( \Delta m_l = \pm 1 \) or 0 \( \Delta j = \pm 1 \) or 0 (total no \\
13. Selection rule for spin:
   "Electric dipole radiation" \( \leftrightarrow \Delta l = 0 \)
   Magnetic dipole or electric quadrupole transitions are forbidden (but do actually occur)
14. Stimulated & spontaneous emission rate \( \propto \vert \mathbf{E} \vert^2 \)
15. \( \mathbf{E} = \mathbf{Q} \times \Delta \mathbf{r} \)
16. Probability of excitation: \( \mathcal{C} = \frac{1}{A + \lambda \mathcal{A} + \ldots} \) \( \mathcal{A} \) are spontaneous emission rates
17. Time-independent 1st order perturbation:
   \( E_n' = E_n + \langle \psi_n^0 | H' | \psi_n^0 \rangle \) \( \psi_n' = \psi_n^0 + \sum_{m
   \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n - E_m} \psi_m^0 \)
18. Quantum approximation of rotational energy: \( E_{rot} = \frac{\hbar^2 L(J)}{2I} \)
19. Electric field: \( E_x = \frac{1}{2} m v^2 \)
20. Differential cross-section: \( d\sigma \propto \frac{\text{scattered flux/unit of solid angle}}{\text{incident flux/unit of surface}} \)
21. Intrinsic magnetic moment: \( \mu = g \frac{e}{2m} \) \( g \) is Landé factor
22. Total cross-section: \( \sigma = \int d\Omega \frac{d\sigma}{d\Omega} \) \( d\Omega \equiv \frac{dS}{S} \)
23. Stark effect is electrical analog to the Zeeman effect
24. Born-Oppenheimer approximation: a reiterative idea
25. In Stern-Gerlach experiment, a beam of neutral silver atoms are sent through an inhomogeneous magnetic field. Classically, nothing happens as the atoms are neutral
with Larmor precession, the beam would be deflected into a smear.
But if it actually deflects into 2x+1 beams, thus coherently with the fact
vectors are of spin $\frac{1}{2}$.

* Know the basic spherical harmonics

$$Y_0 = \frac{i}{\sqrt{2}} Y_{1} = \frac{1}{\sqrt{2}} \sin \theta \cos \phi \ e^{-i\phi} \quad Y_1 = \frac{1}{\sqrt{2}} \cos \theta \quad Y_1(0, \phi) = \frac{1}{\sqrt{2}} \cos \theta \ e^{i\phi}$$

* Probability density current

$$J = \frac{i}{2\pi} \left( \psi \nabla \psi^* - \psi^* \nabla \psi \right) = \text{Re} \left( \psi \nabla \psi^* \right)$$

* Laser operates by going from lower state to high state (population inversion), then falls back on a metastable state in-btw. (not all the way down due to selection rule)

* $\langle P \rangle = \int \psi^* (x, \phi, \theta) \psi(x, \phi, \theta) \ dx$ [X, $\phi$] = i$h$, [X, $\theta$] = i$h \partial \overline{\phi}$

* $m \partial^2 \psi \overline{\partial t}^2 = \frac{\partial^2 \psi}{\partial x^2}$ Ehrenfest's Thm: expectation values obey classical laws

* If $V(x)$ is even, $\psi(x)$ can always be taken to be either even or odd

* $\psi(x) = \frac{1}{\sqrt{N}} \sum_{n=1}^{\infty} C_n e^{i\frac{n\pi}{a} x}$ $\delta(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} \ dx$

* Tunneling shows exponential decay

* The ground state of even potential is even and has no nodes

* In stationary states, all expectation values are independent of t

* Harmonic Oscillators

$$H = \hbar \omega (a \hat{a} - \frac{1}{2}) = \hbar \omega (a + \hat{a} + \frac{1}{2}) \quad C_n = \frac{1}{\sqrt{2^{2n} n! \omega^n}} \left( \frac{\hbar \omega}{\sqrt{2}} \right)^n$$

$$C_{n+1} \frac{\hbar \omega}{2} C_n = \sqrt{n+1} C_n$$

$$A - \hat{a} \psi_n = \hat{a} \psi_n \quad C_{-1} \frac{\hbar \omega}{2} C_0 = \sqrt{2} \psi_0$$

* $\langle \psi \rangle = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} \ dx$

* $\psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-i\frac{p^2}{2\hbar} - ixt} \varphi(x) \ dx$

* $\hat{p} \langle \psi \rangle = \frac{i}{\hbar} \langle [\hat{H}, \psi] \rangle + \langle \psi \hat{p} \psi \rangle$

* Virial Thm. in stationary state $2\langle \dot{\psi} \rangle = \langle \psi \dot{\psi} \rangle$

* Hydrogen Atom revisited

$$E_n \propto \text{reduced mass} \quad \frac{1}{2\hbar^2} \quad En = \frac{1}{2\hbar^2} \left( \frac{e^2}{4\pi \epsilon_0 \hbar c^2} \right)^2 \frac{1}{n^2} = \mu n^2$$

$$\mu = \frac{1}{2} \quad E_n = \frac{1}{2} E_n \quad a(2) = \frac{\alpha}{8} \quad R(2) = \frac{8}{2} R$$
Bohr radius $a = \frac{4\pi \epsilon_0 k^2}{m e^2} = 0.529 \times 10^{-10}$ m

$\Psi_{100}(r, \theta, \phi) = \frac{1}{r^2} e^{-r/a}$

$L_z, L_y \perp \hat{z} \perp L_z \perp \hat{L}, [L_z^2, L_y] = 0, L_z = L_x \pm i L_y, [L_z^2, L_x \pm L_y] = 0$

$L_z f_{L_z} = \alpha \hat{L}(\xi) f_{L_z}, L_z f_{L_z} = \alpha \hat{L} m L_z$

$L_z f_{L_z} = \hbar \xi \hat{L}(\xi m) (1 + \xi m) = \hbar \xi \hat{L} C(\xi m) - \hbar L m(\xi m)

[L_x, L_y] = i \hbar \xi \hat{L}, [L_x, \hat{L} \pm L_y] = \hbar \xi \hat{L} P_y, [L_y, \hat{L} \pm L_x] = -i \hbar P_x$

$L_z = \frac{\hbar}{\xi} \frac{\partial}{\partial \phi}$

$T_y = (0, 0), T_x = \left(\frac{\hbar}{\xi}, 0\right), S = \frac{\hbar}{\xi} \hat{y}$

$\chi_{-} = \left(\frac{\hbar}{\xi}\right) \text{eigenvalue } \frac{\hbar}{\xi}$

$\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle = \frac{\hbar^2}{\xi^2}$

Clebsch-Gordan coefficients

$|S, M_S\rangle = \sum_{m_m, m_m} C_{m,m}^{S, S} |S, m_m\rangle |S, m_m\rangle$

$\sum_{m_m} C_{m,m}^{S, S} |S, m_m\rangle$ continuity equation $\nabla \cdot \mathbf{j} = -\frac{\partial}{\partial t} \left| \psi \right|^2, \int \nabla \cdot \mathbf{J} \, d\mathbf{r} = -\frac{\partial}{\partial t} \int \left| \psi \right|^2 \, d\mathbf{r}$

2st+1 $S = \text{spin (number)} \quad J = \text{total (number)}$

Hund's rule 1) State with highest spin will have lowest energy given Pauli principle satisfied; 2) For given spin and antisymmetrization highest $L$ have lowest energy; 3) No than than half filled, lowest level has $J = L - S$; if more than half-filled $J = L + S$

Fermi Gas $k_B = (3\pi^2)^{\frac{3}{5}}$ $P = \frac{N k_B}{V}$

$P = \frac{3 \pi^2 \frac{3}{5} k_B^2 e^{\frac{3}{5}}}{5 m} \rho$ degeneracy pressure

$n_{(c,e)} = \left\{ \begin{array}{ll} \frac{e^{-\frac{E}{k_B T}}}{e^{-\frac{E}{k_B T}} + 1} & \text{classical} \\ e^{-\frac{E}{k_B T} / h \xi} & \text{Fermion} \\ e^{-\frac{E}{k_B T} / h \xi} & \text{Boson} \end{array} \right.$

Blackbody $P_{(c)} = \frac{2\pi^{\frac{3}{2}} \hbar^3}{r^2 e^{\frac{r^2}{4} c^2 (e^{\frac{r^2}{4} c^2} - 1)}}$
* Fine structure $\Rightarrow$ Spin-orbit coupling
  
  Relative correction $\Delta = \frac{1}{137.036}$

  Then, Lamb shift electric field

  Then, Hyperfine structure due to magnetic interaction between electrons and protons

  Spin-spin coupling (21 cm line)

* Fine structure breaks degeneracy in $l$ but still have $j$

* Fermi's golden rule is a way to calculate the transition rate (probability of transition per unit time) from one energy eigenstate of a quantum system into a continuum of energy eigenstates, due to a perturbation

* Full shell & close to full shell config are more difficult to ionize

* Larmor precession $\vec{\pi} = \mu \vec{B}$

  $\omega = \gamma B$
VI ATOMIC PHYSICS

* $\Delta E = h f = h \frac{\lambda}{c}$, $h c = (2.4 \text{ keV} \cdot \AA = 12.90 \text{ eV} \cdot \text{nm}$, de Broglie wavelength $\lambda = \frac{h}{mv}$

* Emission due to transition from level $n$ to level $m$

  $\frac{1}{\lambda} = R \left( \frac{1}{a^2} - \frac{1}{n^2} \right)$  $a=1$ Lyman series, $a=2$ Balmer series

  $R = 1.097 \times 10^7 \text{ m}^{-1}$ (not given in the table), $E_n = -\frac{13.6 \text{ eV}}{n^2}$

* Hydrogen model extended, $R = n^2 \cdot \text{ proton, quantities scale as}\ E \sim n^2$, $\lambda \sim \frac{1}{n^2}$

  reduced mass correction to emission formula is $\frac{1}{\lambda} = \frac{R a^2}{1 + (a/n)} \left( \frac{1}{a^2} - \frac{1}{n^2} \right)$

  $m =$ mass of electron, $M =$ mass of proton $m/M = 1/1836$

* Bohr postulate $L = mv \times r = \frac{n \hbar}{2\pi} = n \hbar$

* Zeeman effect: splitting of a spectral line into several components in the presence of a static magnetic field.

* $k$ series refers to the inner-most shell (K, L, M, N) so transition to inner-most shell $E = -13.6 \text{ eV} \left( 1 - \frac{1}{n^2} \right) \text{ eV}$

  shielding

* Frank-Hertz Expt: Electrons of a certain energy range can be scattered inelastically, and the energy lost by electrons is discrete.
E² = (pc² + (mc²)²) for photon (massless particle) \( E = pc = hv \)

* Relativistic Doppler Effect \( \lambda = \frac{\lambda_0}{\sqrt{1 - \beta}} \) \( \beta = \frac{v}{c} \)

Sign can be determined by whether it is moving away/closer

* Space-time interval \( \Delta s = (\Delta t² - \Delta x² - \Delta y² - \Delta z²)^{\frac{1}{2}} \)

* Lorentz transformation

\[
ct' = \gamma (ct - bx), x' = \gamma (x - \beta ct), y' = y, z' = z
\]

* Relativistic addition of velocities

\[
\frac{U_x'}{\gamma} = \frac{U_x + \beta v}{\gamma (1 + \beta v/c²)}, \quad \frac{U_y'}{\gamma} = \frac{U_y}{\gamma (1 + \beta v/c²)}, \quad \frac{U_z'}{\gamma} = \frac{U_z}{\gamma (1 + \beta v/c²)}
\]

* Lorentz-Transformation of EMF, \( E_x' = E_x - \beta (E_y + B z) \) \( E_y' = E_y - \beta (E_z + B_x) \) \( E_z' = E_z - \beta (E_x + B_y) \)

* \( E = \rho m c², \quad P = \rho m v \)

* In every closed system, the total relativistic energy and momentum are conserved

* Spacelike separation, i.e. can happen at same time \( \Delta t² - \Delta x² < 0 \)

* Transverse Doppler shift \( f = \frac{f_0}{\sqrt{1 - \beta²}} \) or \( f = f'_0 \sqrt{1 - \beta²} \)
VIII. LABORATORY METHODS

* If measures are independent (or intervals in a poisson process are independent), both expected value and variance increase linearly with time (in this case), so longer time can improve uncertainty (which is usually defined as $\frac{1}{\sqrt{T}} \propto \frac{1}{\sqrt{N}}$).

* In Poisson distribution $\sigma = \sqrt{X}$, so $\sigma$ square root of the average.

* Error Analysis: Estimating uncertainties

  If you are sure the value is close to 26 mm than to 25 or 27 mm, then record best estimate $36 \pm 0.5$ mm.

* Propagation of uncertainties

  For sum of random and independent variables

  $$\delta X = \sqrt{(\delta X_1)^2 + (\delta X_2)^2 + (\delta X_3)^2 + \ldots}$$

  If multiplication or divisions are involved, use fractional uncertainty

  $$\frac{\delta a}{a} = \sqrt{(\frac{\delta X_1}{X_1})^2 + (\frac{\delta X_2}{X_2})^2 + \ldots}$$

* Experimental uncertainties that can be revealed by repeating the measurements are called random errors; those that cannot be revealed in this way are called systematic errors.
IX SPECIALIZED TOPICS

* Photoelectric effect \( E_{\text{photon}} = \text{work function} + KE_{\text{max}} \)

* Compton Scattering \( \lambda' \rightarrow \lambda' - \frac{h}{mc} \left( 1 - \cos \theta \right) \)

  \( \frac{h}{mc} \) is Compton wavelength

* X-Ray Bragg reflection \( n\lambda = 2d\sin \theta \) (compare to diffraction grating \( n\lambda = d\sin \theta \))

* 1.602 x 10^{-19} \text{J} = (1.602 x 10^{-9} \text{c}) (1 \text{eV}) = 1 \text{eV}

* In solid-state physics effective mass \( m^* = \frac{\hbar^2}{d^2} \frac{\partial^2 E}{\partial k^2} \)

* Electronic filters

  ![filter diagram]

  In. → Out

  High-pass means \( f \rightarrow 0 \), \( V_{in} = V_{out} \)

  Usually look at \( I = \frac{V_{in}}{Z} \) \( Z = R + j(X_L - X_C) \)

  \( X_L \approx \omega L \), \( X_C \approx \frac{1}{\omega C} \)

* Band spectra is a term that refers to using EM waves to probe molecules.

* Solid state: primitive cell = \# of lattice points in a Breswol lattice

  - simple cubic \( \rightarrow 1 \) point
  - body centered \( \rightarrow 2 \) points
  - face centered \( \rightarrow 4 \) points

* Resistivity of undoped semiconductor varies as \( 1/T \)

* Nuclear physics: binding energy is a form of potential energy, is to take it as positive. It's the energy needed to separate into separate different constituents. It is usually subtracted from other energy to tally total energy.

* Pair production refers to the creation of an elementary particle and its antiparticle. Usually need high energy (at least the total rest mass)

* At low energies, photoelectric effect dominates Compton Scattering

* Radioactivity: beta decay \( X^A_\beta \rightarrow X'^A_\beta^- + e^+ + v \)

  - Alpha: \( X^A_\beta \rightarrow X'^A_\beta^+ + 4He^+ \)
  - Gamma: \( X^A_\beta \rightarrow X^A + \gamma \)

  - Deuteron Decay (not natural) \( X^A_\beta \rightarrow X^A-2 + H^+ \)

  Radioactivity usually follows Poisson

* Coaxial cable terminated at one end with characteristic impedance in order to avoid reflection of signals from the terminated end of cable.

* Human eyes can only see things in motion up to \( \sim 25 \text{Hz} \)

* In magnetic field, electrons are more likely to be emitted in a direction opposite to the spin direction of the decaying atom.
× Opamp (operational amplifiers): If you only have two days to prepare for the GRE, this is not worth the effort. Maximum one question on this. It is interesting read nonetheless. Recommend "The Art of Electronics".

× The specific heat of a superconductor jumps to a lower value at the critical temperature (resistivity jumps too)

× Elementary particles:

\[
\begin{align*}
\text{Quarks} & : \quad \uparrow \text{up}, \quad \downarrow \text{down}, \quad \uparrow \text{top}, \quad \downarrow \text{bottom}, \quad \text{gluon} \\
\text{Electrons} & : \quad \text{Leptons} \\
\text{Mesons} & : \quad \text{Hadrons} (\text{bound state of quarks})
\end{align*}
\]

- Baryons
- 3 quarks form
- Fermion
- (Baryon: $N$, $\Delta$, $\Lambda$)
- $N$ (nucleon)
- $\Delta$, $\Lambda$ usually spin $\frac{1}{2}$

- Mesons
- quark-antiquark pair
- form bosons
- Baryon no. $B = 0$
- e.g., $\pi$, $K$, $\Lambda$, (kaon)

- Leptons
- $e$, $\mu$, $\tau$, $\nu_e$, $\nu_\mu$, $\nu_\tau$, $\nu_	ext{tau}$
- $W$, $Z$, $\nu$ boson

- Generations
- $I$, $II$, $III$

- Fermions

- Family no. is preserved
- * lepton no. conserved
- (\# of leptons = \# of antileptons)
- * strange ness is conserved except for weak interactions
- *(S = -C_{N3} - N_{\overline{3}})* strange antiparticle
- * Baryon no. conserved
- $B = \frac{N_2 - N_\overline{2}}{3}$

× Internal conversion is a radioactive decay where an excited nucleus interacts with an electron in one of the long electron shells, causing the electron to be emitted from the atom. It is not Beta decay.