

# Postulates

**Invariance of physical laws in all inertial frames.**

**Constancy of the speed of light  $c$  in a vacuum.**

# Helpful Equations

## Lorentz Transformation

Let  $S$  denote the frame in which we have an event (spacetime point) with coordinates  $(t, x, y, z)$ .

Then let  $S'$  denote the frame in which the coordinates of the event are  $(t', x', y', z')$ , and which is traveling with a velocity  $v$  with respect to  $S$  along the  $x$ -axis:

$$\begin{aligned}t' &= \gamma \left( t - \frac{vx}{c^2} \right) \\x' &= \gamma(x - vt) \\y' &= y \\z' &= z\end{aligned}$$

$\gamma$  is the Lorentz factor. Note that as  $v \rightarrow 0$ ,  $\gamma \rightarrow 1$  (Galilean limit), and as  $v \rightarrow c$ ,  $\gamma \rightarrow \infty$ :

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Sometimes people write  $\beta = v/c$ , then  $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$ .

A sometimes helpful invariant:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

The interval between two events is said to be spacelike if  $(\Delta s)^2 < 0$ , timelike if  $(\Delta s)^2 > 0$  and lightlike if  $(\Delta s)^2 = 0$ . There can be a cause-effect relationship between two points if and only if the spacetime interval is timelike or (if the event is a photon traveling between two points) lightlike.

## Velocity addition

If an object appears to travel at velocity  $w$  (parallel to  $v$ ) in  $S$ , then it appears to travel at velocity  $w' = \frac{w-v}{1-wv/c^2}$  in  $S'$ .

## Dynamics

Let  $m_0$  be the rest mass of an object. Then if it is moving at  $\vec{v}$  in  $S$ , then its energy is:

$$E = \gamma m_0 c^2$$

and its momentum is:

$$\vec{p} = \gamma m_0 \vec{v}$$

Furthermore,  $E^2 - |pc|^2 = (m_0 c^2)^2$ , which is invariant. Also,  $F = \frac{dp}{dt}$  instead of  $ma$ .

## Time Dilation / Length Contraction

An object has length  $L$  in  $S$ , and a time interval has length  $T$  in  $S$ . Then in  $S'$ :

$$\begin{aligned} L' &= L/\gamma = L\sqrt{1 - v^2/c^2} \\ T' &= \gamma T = \frac{T}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

The length is contracted and the time is dilated.